

KERR-LENS MODE LOCKING IN A RING BIDIRECTIONAL YAG:CR4+ LASER

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Abstract

This paper concerns a bidirectional ring YAG:Cr⁴⁺ laser. We investigated the features of bidirectional generation and explored the possibility of gyroscopic effect in the laser. We developed the model of bidirectional generation of such a laser taking into account spectral properties of the active medium.

Experiments in our setup revealed that generation of the laser had a cluster character. Luminescence spectra of the laser showed that this medium has both homogeneous and inhomogeneous broadenings. Stable bidirectional generation is a result of the fact that competition between counterpropagating waves is less than in the case of homogeneously broadened medium.

Introduction

Kerr-lens mode-locked lasers with broadband active media (Al₂O₃:Ti³⁺, Mg₂SiO₄:Cr⁴⁺, YAG:Cr⁴⁺) are usually used as sources of femtosecond pulses with tunable frequencies. Laser gyros with such active media have a large potential due to their fabricability, generation effectiveness, mechanical reliability and high accuracy. The majority of solid-state active media has homogeneous broadening of laser transition that results in a strong competition between counterpropagating waves and complexity of generation dynamics [1, 2] in ring lasers with such active media. But in ring lasers with broadband active media Kerr-lens mode locking with femtosecond pulse generation can be achieved, and the wave competition and lock-in effect can be reduced essentially. This indicates that lasers with broadband active media have good prospects in laser gyroscopy.

Ring laser used as a laser gyro must have stable bidirectional generation and (in the case when mode locking is used to reduce the lock-in zone) stable mode locking regime. To improve the stability and to reduce the level of fluctuations of the laser it is necessary to carry out the calculations with taking into consideration the spectral characteristics of the laser transition and the broad amplification line.

In this work we investigated how the spectral characteristics of laser transition of the broadband active medium YAG:Cr⁴⁺ affect the level of fluctuations and stability of bidirectional generation, and stability of mode locking in a ring laser.

Broad amplification profile of YAG:Cr⁴⁺ (fig.1-2) allows to obtain femtosecond pulse generation by phase locking of a great number of longitudinal modes (fig.2).

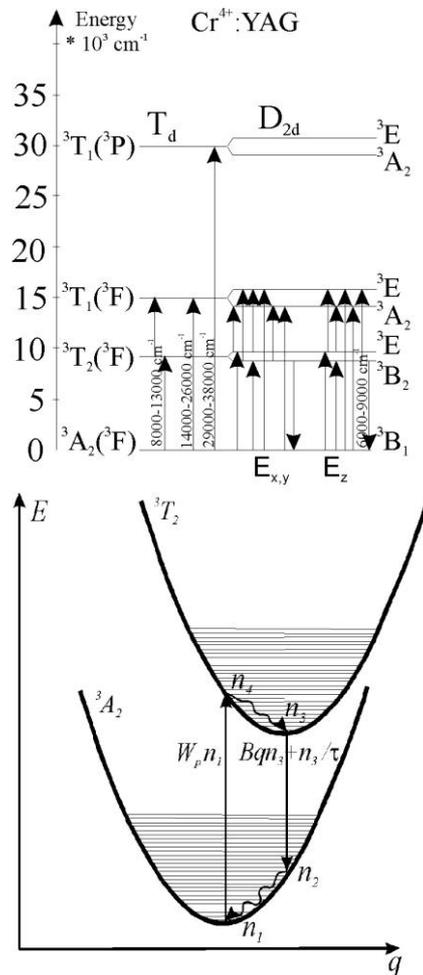


Fig.1. Energy level diagram for YAG:Cr⁴⁺ and for vibrational transition laser.

According to a number of papers [3], longitudinal mode locking can enhance gyroscope accuracy due to the decrease of natural width of line (fig.2). Meanwhile the competition between counterpropagating waves in the ring laser with locked modes becomes less as the synchronizing

pulses traverse the gain medium by turns and interaction between them decreases significantly.

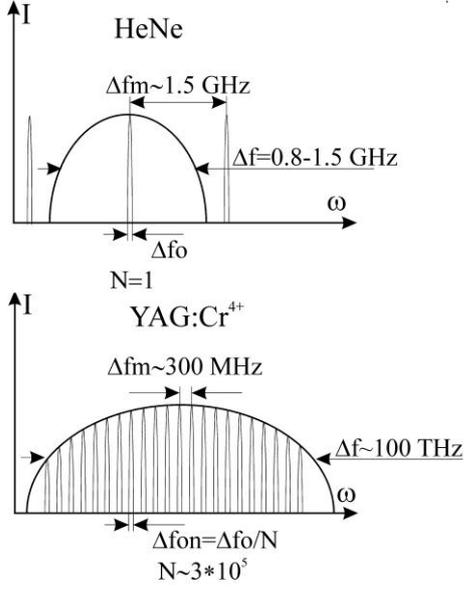


Fig.2. Longitudinal mode spectrum of He-Ne ring laser and YAG:Cr⁴⁺ ring laser.

In order to determine the strength of interaction between the counterpropagating waves and between the longitudinal modes we performed the numerical simulation and analytical calculation of the laser parameters.

Numerical Simulation

To model the bidirectional generation in case of multifrequency generation in a ring laser we used the semiclassical self-consistent approach. As a starting point of the study we used Maxwell wave equation for electric field in the cavity (transverse structure was neglected) [2, 4]:

$$\nabla^2 \vec{E} - \text{grad}(\text{div} \vec{E}) = \frac{4\pi\sigma}{c^2} \frac{\partial \vec{E}}{\partial t} + \frac{\varepsilon}{c^2} \frac{\partial}{\partial t^2} \left(\vec{E} + \frac{4\pi\vec{P}}{\varepsilon} \right), \quad (1)$$

where σ is the conductivity associated with volume losses in the medium (losses are modulated in intermode beat frequency to make modes be locked), ε is the permittivity of the medium.

The field in the cavity is the superposition of fields of counterpropagating waves that can be expanded into longitudinal modes:

$$\vec{E} = \frac{1}{2} \sum_{j=1}^J (\vec{E}_j^+ e^{-ik_j x} + \vec{E}_j^- e^{ik_j x}) e^{i\omega_j t}. \quad (2)$$

The laser frequency was assumed to be considerably greater than intermode beat frequency. Interaction between different longitudinal modes through inverse population gratings was neglected. Longitudinal mode locking can be achieved by loss modulation: every longitudinal mode has two sidebands, and if the modulation frequency is the same as the cavity-mode spacing, these sidebands

correspond to the two cavity modes adjacent to the original mode, and the modes become phase-locked. For a first approximation we assume that the frequency differences caused by gyroscope rotation or by any other type of non-reciprocity are the same for all modes.

In order to find the polarization entering in (1) we solved the material equations for evolution of the density matrix [4-6] of solid-state broadband medium. In case of homogeneously broadened medium contributions of all the active ions to polarization are the same, so to determine the polarization one only need to know the density of active centers and the elements of dipole moment matrix, and to calculate the elements of density matrix. To find the polarization in case of inhomogeneously broadened medium one should sum the contributions of different groups of centers according to the spectral distribution. Therefore to find the polarization we must sum the contributions of groups of centers with various operating frequencies ω_o according to the distribution

$h(\omega_o)$:

$$\vec{P}_{mn} = N_s \int_0^\infty (\rho_{nm} \vec{d}_{nm} + \rho_{nm} \vec{d}_{nm}^*) d\omega_o, \quad (3)$$

$$m, n = \overline{1, 2},$$

taking into account the normalizing conditions:

$$\int_0^\infty h(\omega_o) d\omega_o = 1, \quad (4)$$

$$\rho_{11}(\omega_o) + \rho_{22}(\omega_o) = h(\omega_o).$$

Here N_s is a density of the active centers, ρ_{mn} are density matrix elements, \vec{d}_{nm} are dipole matrix elements, n, m are level numbers. We represent the spectrum of our active medium as sum of contributions of several groups of the homogeneously broadened centers.

The elements of density matrix can be determined from the quantum kinetic equations [4-6] in two level approximation with taking into account the pump, relaxation transitions between the levels and the relaxation of the off-diagonal elements of the density matrix:

$$\frac{d\rho_{mn}}{dt} + (\gamma_{nm} + i\omega_o^{mn})\rho_{mn} = \frac{i}{\hbar} \vec{E} \sum_q (\vec{d}_{mq} \rho_{qn} - \vec{d}_{qn} \rho_{mq}), \quad (5)$$

$$\frac{d\rho_{mm}}{dt} + \sum_q (\omega_{mq} \rho_{mm} - \omega_{qm} \rho_{qq}) = \frac{i}{\hbar} \vec{E} \sum_q (\vec{d}_{mq} \rho_{qm} - \vec{d}_{qm} \rho_{mq}).$$

Here γ_{nm} is the rate of relaxation of the off-diagonal elements of the density matrix, ω_{nm} is the probability of relaxation transition, ω_o^{mn} is the operating frequency, \vec{E} is the field of radiation.

For homogeneously broadened active medium one can derive from (2) equations for polarization of the medium and inverse population of the active ions excited by laser field [4-6]:

$$\begin{aligned} \frac{\partial^2 \bar{P}_{ak}(\bar{r}, t)}{\partial t^2} + \frac{2}{T_2} \frac{\partial \bar{P}_{ak}(\bar{r}, t)}{\partial t} + \left(\omega_o^2 + \frac{1}{T_2^2} \right) \bar{P}_{ak}(\bar{r}, t) = \\ = -\frac{2\omega_o}{\hbar} |\bar{d}|^2 N(\bar{r}, t) \bar{E}(\bar{r}, t) \\ \frac{\partial N(\bar{r}, t)}{\partial t} = -\frac{N(\bar{r}, t) - N_{\text{max}}(\bar{r}, t)}{T_1} + \frac{2}{\hbar \omega_o} \left(\bar{E}(\bar{r}, t), \frac{\partial \bar{P}_{ak}(\bar{r}, t)}{\partial t} \right) \end{aligned} \quad (6)$$

Here $\bar{E}(\bar{r}, t)$ is a field of the wave; $\bar{P}_{ak}(\bar{r}, t)$ is a polarization of the active particles; $N(\bar{r}, t)$ is an inverse population; $N_{\text{max}}(\bar{r}, t)$ is a part of the inverse population created by the pump; T_1 is a longitudinal relaxation time; T_2 is a transverse relaxation time; $|\bar{d}|^2$ is a squared magnitude of the dipole moment; ω_o is a frequency of laser transition.

One can see some results of the simulation for the medium with both homogeneous and unhomogeneous broadenings at fig. 3 and fig. 4. The polarization of the medium was expressed as a sum of the contributions of groups of centers with different frequencies.

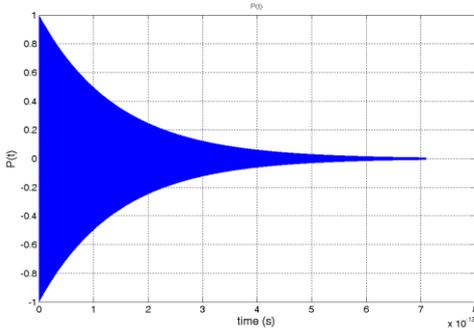


Fig.3. Polarizability evolution.

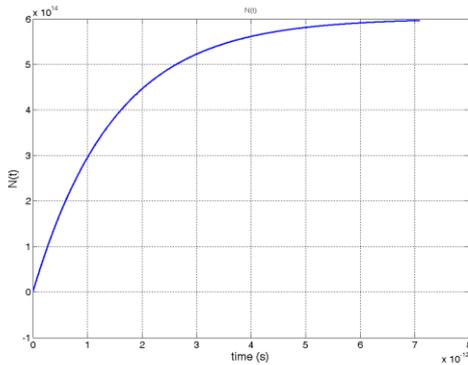


Fig.4. Evolution of population inversion.

In case of multifrequency generation and homogeneous broadenings we have the following reduced equations for the slowly-varying amplitudes of counterpropagating fields and for the population inversion:

$$\begin{cases} \frac{d\tilde{E}_j^\pm}{dt} = -\frac{\omega_j}{2Q_j} \tilde{E}_j^\pm + \frac{i}{2} \tilde{m}_j^\pm \tilde{E}_j^\mp + \left[\sum_{k \neq j} \frac{i}{2} \tilde{d}_k^\pm \tilde{E}_k^\pm \right] \mp \frac{i}{2} \Omega \tilde{E}_j^\mp + \\ + \left[\frac{i}{2} \Delta\omega_r \tilde{E}_j^\pm + \frac{\sigma_j c}{2L} \left(\tilde{E}_j^\pm \int_0^L N dx + \tilde{E}_j^\mp \int_0^L N e^{\pm 2i(k_j + \Delta k_r)x} dx \right) \right], \\ \frac{dN}{dt} = W - \frac{N}{T_1} - \frac{N}{T_1} \sum_j a_j \left| \tilde{E}_j^+ e^{-i(k_j + \Delta k_r)x} + \tilde{E}_j^- e^{i(k_j + \Delta k_r)x} \right|^2, \\ j = \overline{1, J}. \end{cases} \quad (7)$$

Here j is a longitudinal mode number, \tilde{m}_j^\pm are the complex coupling coefficients of counterpropagating waves, \tilde{d}_j^\pm are the complex coupling coefficients of longitudinal modes, T_1 is the upper-state relaxation time, W is the pumping rate, L is the cavity length, l is the length of the crystal, σ_j is the transition cross-section for the mode j , $a_j = \sigma_j c T_1 / 4h\omega$ is the saturation parameter, Ω is the angular frequency non-reciprocity, Q_j is the cavity figure of merit, $k_j = 2\pi m_j / L$ is a wave number, $\Delta\omega_r$ is the fluctuations of cavity frequency, Δk_r is the fluctuations of wave number.

Boundary conditions for the wave equation:

$$\tilde{E}(x + L + \Delta L_r, t) = \tilde{E}(x, t), \quad (8)$$

where ΔL_r - random fluctuations of the perimeter.

So the fluctuations of wave number are given by

$$\Delta k_r \approx \frac{\partial k_j}{\partial n_j} \Big|_{n=n_j; L=L_0} dn_j + \frac{\partial k_j}{\partial L} \Big|_{n=n_j; L=L_0} dL. \quad (9)$$

The beat frequency of the counterpropagating waves is given by

$$\begin{aligned} \tilde{E}_j^\pm = \left| \tilde{E}_j^\pm \right| e^{i\phi_j^\pm(t)}, \\ \Delta\omega_j = \omega_j^+ - \omega_j^- = \frac{d\phi_j^+}{dt} - \frac{d\phi_j^-}{dt}. \end{aligned} \quad (10)$$

Some results of numerical simulation for the laser with fluctuating resonator parameters are presented at fig.5-8 (two levels of fluctuations are concerned). The synchronous fluctuations of these parameters reduce the amplitude of dynamic gratings of population inversion (fig. 5, 6), and interaction of counterpropagating waves on these gratings becomes smaller.

Experimental Setup

We used an enhanced experimental setup [8] (fig.9). It enabled us to study the features of bidirectional generation and to explore the possibility of gyroscopic effect in a ring laser with a broadband solid-state active medium. Setup is based on the ring YAG:Cr⁴⁺ laser pumping by YAG:Nd³⁺ or by ytterbium fiber laser. It also includes the systems that allow us to create frequency non-reciprocity, realize fine wavelength tuning and stabilize the temperature of the active medium.

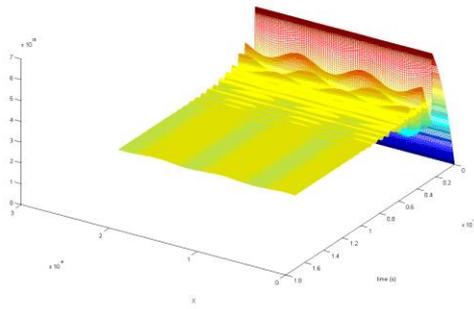


Fig.5. Evolution of the dynamic grating of population inversion in the laser with very low fluctuations of resonator parameters. One can see a weak smoothing of the grating.

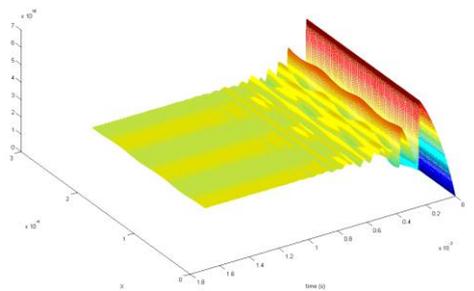


Fig.6. Evolution of the dynamic grating of population inversion in case of fluctuations of resonator parameters with frequency variation range comparable to the lock-in zone. One can see a stronger smoothing of the grating.

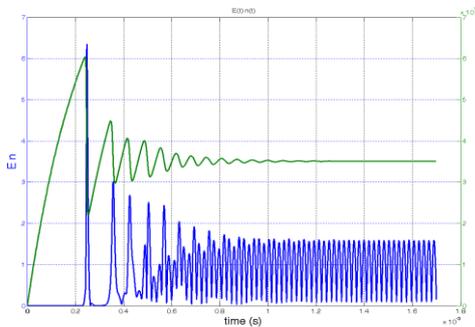


Fig.7. Evolution of population inversion and field amplitude inversion in the laser with very low fluctuations of resonator parameters. Self-modulation oscillations.

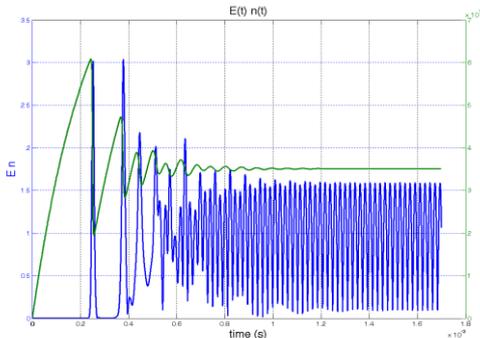


Fig.8. Evolution of population inversion and field amplitude inversion in case of fluctuations of resonator parameters with frequency variation range comparable to the lock-in zone. Self-modulation oscillations and nonregular oscillations of the inversion.

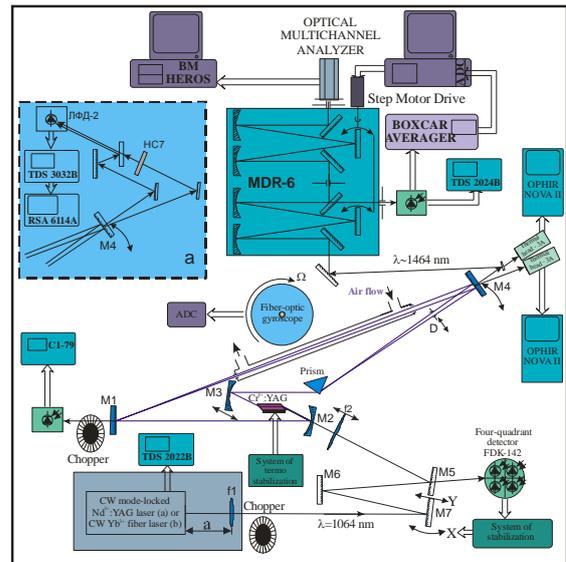


Fig.9. Experimental setup; a – scheme for observing the beats of counterpropagating waves.

The laser resonator has a Z-configuration, includes two spherical and two flat mirrors and a quartz prism. Wavelength and spectrum width can be changed by turn of the mirrors M3 and M4 and by variation of width and position of the aperture D. A 20-mm long YAG:Cr⁴⁺ crystal rod was used as an active element. The crystal was 6 mm in diameter with faces oriented under Brewster angle. Cr⁴⁺ ion concentration is $5 \times 10^{17} \text{ cm}^{-3}$. The characteristic parameters of the crystal were described in [9]: the upper-state relaxation time is 3.6 μs , the transition cross-section is $(7-8) \times 10^{-19} \text{ cm}^2$ for 1.42 μm . The center of the absorption line is about 1 μm that corresponds to ${}^3B_1({}^3A_2) \rightarrow {}^3A_2({}^3T_1)$ transition. The absorption factor for 1.06 μm is 2.5 cm^{-1} . So absorption due to the length of the crystal is about 90%.

At the laser output we set an optical mixer bringing waves together to obtain beat note signal. In order to create the frequency nonreciprocity we used Fizeau-Fresnel effect and Faraday effect obtained by the application of an external magnetic field to the active medium.

Experimental Results

We obtained bidirectional generation at the laser. The output laser power in bidirectional regime is 200 mW. CW-CCW intensity ratio can be varied from 1:1 to 1:3 (fig.10). It is one of the main problems of ring solid-state lasers to obtain bidirectional generation because of strong interaction of counterpropagating waves; unidirectional regime can be achieved easier. But we obtained nearly equal intensities of the waves, so the beat note regime can be realized in our laser.

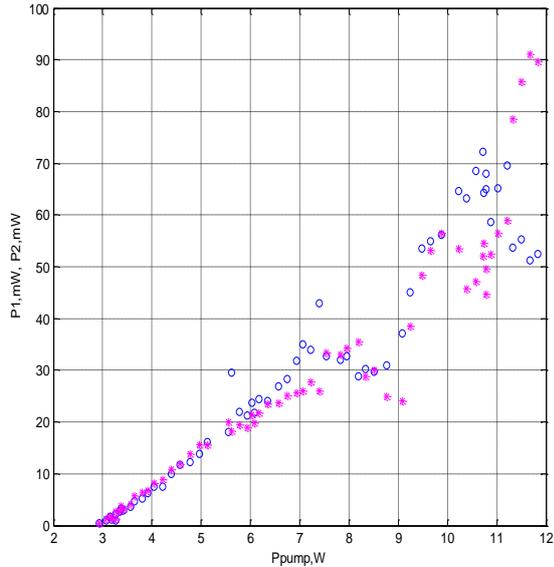


Fig.10. CW and CCW outputs as functions of pump power.

We measured the luminescence spectra of the active medium (fig.11, 12) in the laser generated radiation of various frequencies and intensities and also in the absence of generation. On these spectra one can see a small intensity decay only at the short-wave tail. This fact indicates that laser generation has a small influence on the gain factor at the neighbouring frequencies, and the competition between the counterpropagating waves is low.

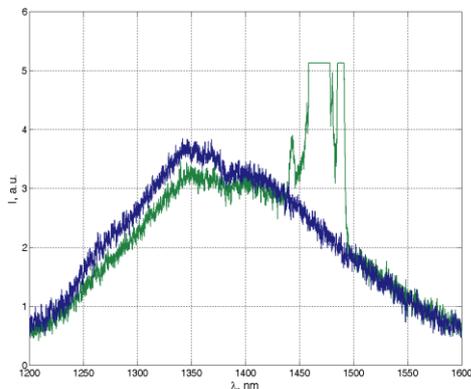


Fig.11. Luminescence spectra of the active medium in the operating laser (chopped line, $\lambda_{gen}=1465$ nm) and in the absence of generation.

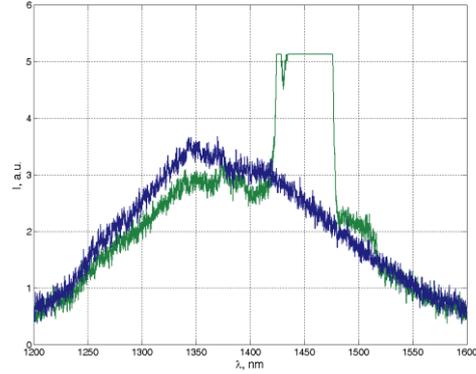


Fig.12. Luminescence spectra of the active medium in the operating laser (chopped line, $\lambda_{gen}=1450$ nm) and in the absence of generation.

There are some clusters in the laser generation (fig.12); that can be caused by the presence of the groups of active centers with different transition frequencies in the gain medium.

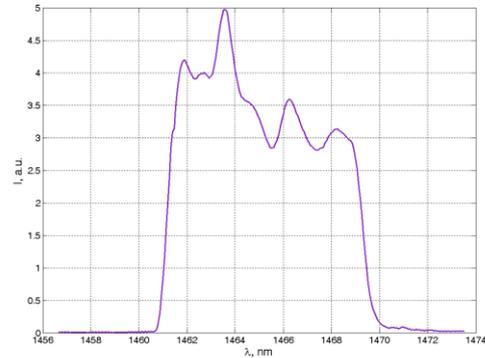


Fig. 12. Generation spectra.

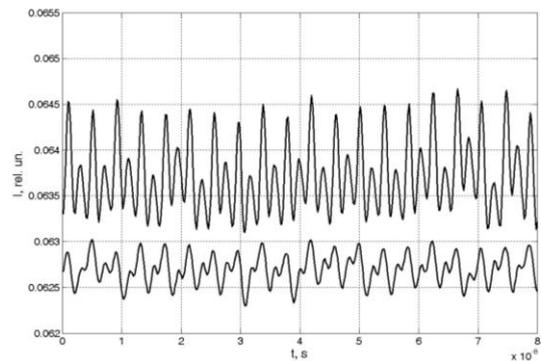


Fig. 14. Kerr-lens mode locking.

Conclusion

We developed the mathematical model for dynamics of bidirectional generation in a ring multifrequency solid-state laser on a broadband active medium with both homogeneous and unhomogeneous broadenings. The polarization of the medium was represented as a sum of the contributions of different groups of centers with various transition frequencies. The model was supplemented by taking into account the noise instability of cavity parameters. Numerical

simulation showed the smoothing of the inversion population gratings due to the fluctuations of laser parameters.

As a result of the calculations we constructed the experimental setup based on the ring YAG:Cr⁴⁺ laser; Kerr-lens mode-locked operation of the laser was obtained (fig.14). We investigated the influence of the magnitude of non-reciprocity on the laser dynamics in the mode-locked regime. In our laser it is possible to use a small number of intracavity elements to obtain the soliton operation only due to material dispersion and Kerr-type non-linearity. We also considered the problem of rotation sensing by determination of the group velocity delay of the pulses in the ring laser.

Our investigations show that because of spectral characteristics of broadband active medium the competition between counterpropagating waves in the ring YAG:Cr⁴⁺ laser is small so the ring laser with this active medium has prospects for application as a laser gyroscope.

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