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HOW PROPOSITIONS BEGIN
Towards an Interpretation of ὑπόθεσις in Plato's *Divided Line*

The early meaning of the Greek word ὑπόθεσις is a familiar puzzle, with two main pieces of evidence: the Hippocratic treatise *On Ancient Medicine*[1] and Plato's discussion of mathematics (and, in extension, of dialectic) in and around the *Divided Line* passage in the *Republic*. The aim of this article is to suggest a new approach to the puzzle. In the absence of new texts or new linguistic evidence, the line of inquiry is based on the question: what was the mathematical practice familiar to Plato? I concentrate on the usage in the *Divided Line* passage alone, and the aim of this paper is not to offer a general theory of the usage of the term in Plato's time but rather an account of Plato's intention in using the term in this passage.

1. THE TEXT AND SOME POSSIBLE INTERPRETATIONS

The central text for Plato's use of the word ὑπόθεσις is the *Divided Line* passage, 510 c 1 – 511 a 1. I despair of providing an interpretation-free translation of the text and instead take as my starting point the Greek text, followed by two English translations: one relatively free of specific interpretation, the second very rich in interpretation. (In both, however, I take the liberty of re-instating transliterated *hypothesis* where the translation has some English equivalent.) While the second is much less literal, it also remains, in my view, the finest homage to Plato's *Republic* in the English language, and I shall mostly use Cornford's version as my base text.

> [ΣΩ.] Ἀλλ' αὖθις ... ῥᾶον γὰρ τούτων προειρημένων μαθήσῃ. οἶμαι γάρ σε εἰδέναι ὅτι οἱ περὶ τὰς γεωμετρίας τε καὶ λογισμοὺς καὶ τὰ τοιαῦτα πραγματευόμενοι, ὑποθέμενοι τό τε περιττὸν καὶ τὸ ἄρτιον καὶ τὰ σχήματα καὶ γωνιῶν τριττὰ εἴδη καὶ ἄλλα τούτων ἀδελφὰ καθ' ἑκάστην μέθοδον, ταῦτα μὲν ὡς εἰδότες, ποιησάμενοι ὑποθέσεις αὐτά, οὐδένα λόγον οὔτε αὐτοῖς οὔτε ἄλλοις ἔτι ἀξιοῦσι περὶ αὐτῶν διδόναι ὡς παντὶ φανερῶν, ἐκ τούτων δ' ἀρχόμενοι τὰ λοιπὰ ἤδη

[1] The best study remains G. E. R. Lloyd, "Who is Attacked in *On Ancient Medicine*?", *Phronesis* 8 (1963) 108 – 126; reprinted in his *Methods and Problems in Greek Science* (Cambridge 1991) 49 – 69 with some more recent literature.

διεξιόντες τελευτῶσιν ὁμολογουμένως ἐπὶ τοῦτο οὗ ἂν ἐπὶ σκέψιν ὁρμήσωσι.
[ΓΛ.] Πάνυ μὲν οὖν ... τοῦτό γε οἶδα.
[ΣΩ.] Οὐκοῦν καὶ ὅτι τοῖς ὁρωμένοις εἴδεσι προσχρῶνται καὶ τοὺς λόγους περὶ αὐτῶν ποιοῦνται, οὐ περὶ τούτων διανοούμενοι, ἀλλ᾽ ἐκείνων πέρι οἷς ταῦτα ἔοικε, τοῦ τετραγώνου αὐτοῦ ἕνεκα τοὺς λόγους ποιούμενοι καὶ διαμέτρου αὐτῆς, ἀλλ᾽ οὐ ταύτης ἣν γράφουσιν, καὶ τἆλλα οὕτως, αὐτὰ μὲν ταῦτα ἃ πλάττουσίν τε καὶ γράφουσιν, ὧν καὶ σκιαὶ καὶ ἐν ὕδασιν εἰκόνες εἰσίν, τούτοις μὲν ὡς εἰκόσιν αὖ χρώμενοι, ζητοῦντες δὲ αὐτὰ ἐκεῖνα ἰδεῖν ἃ οὐκ ἂν ἄλλως ἴδοι τις ἢ τῇ διανοίᾳ.

Grube:[2]

[Socrates]: Let's try again. You'll understand it more easily after this explanation. I think you know that students of geometry, calculation, and the like *hypothemenoi* the odd and the even, the various figures, the three kinds of angles, and other things akin to these in each of their investigations, as if they were known. They make these their *hypotheseis* and don't think it necessary to give any account of them, either to themselves or to others, as if they were clear to everyone. And going from these *hypotheseis* through the remaining steps, they arrive validly at a conclusion about which they set out to investigate.
[Glaucon:] I certainly know that much.
[Socrates:] Then you also know that, although they use visible figures and talk about them, their thought isn't directed to these but to those other things that they are like. The claims they make are about the square itself and the diagonal itself, not about the diagonal they draw, and similarly with the others. These figures that they make and draw, of which shadows and reflections in water are images, they now in turn use as images, in seeking to see those others themselves that one cannot see except by means of thought.

Cornford:[3]

[Socrates:] Then we will try again; what I have just said will help you understand. You know, of course, how students of subjects like geometry and arithmetic begin *hypothemenoi* odd and even numbers, or the various figures and the three kinds of angle, and other such data in each subject. These data they take as known; and, having adopted them as *hypotheseis*, they do not feel called upon to give any account of them to themselves or to anyone else, but treat them as self-evident. Then, starting from these *hypotheseis*, they go on until they arrive, by a series of consistent steps, at all the conclusions they set out to investigate.

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[2] Plato, *Republic*, tr. by G. M. A. Grube, rev. by C. D. C. Reeve (Indianapolis 1992) 184.

[3] *The Republic of Plato*, tr. by F. M. Cornford (Oxford 1941) 220.

> [Glaucon:] Yes, I know that.
> [Socrates:] You also know how they make use of visible figures and discourse about them, though what they really have in mind is the originals of which these figures are images: they are not reasoning, for instance, about this particular square and diagonal which they have drawn, but about *the* square and *the* diagonal; and so in all cases. The diagrams they draw and the models they make are actual things, which may have their shadows or images in water; but now they serve in their turn as images, while the student is seeking to behold those realities which only thought can apprehend.

This text falls into two sections (to be called "section 1" and "section 2", respectively). A part of my argument later on will depend on the question of the relationship between the two sections.

We are primarily interested in the question, what are mathematicians said to do in section 1. Jowett, for instance, had the following note:[4] "...they presuppose mathematical quantities and figures without any inquiry into the grounds of their suppositions, and end... ὁμολογουμένως – i.e., consistently..." Two things present in mathematical activity, according to Jowett, are the "presuppositions of quantities and figures" and "consistency". One thing missing is "an inquiry into the grounds of the presuppositions".

It is not altogether clear what is meant by "presupposing a quantity or a figure". Plato indeed says that the mathematicians treat as ὑποθέσεις the odd and the even, for instance, so it would seem that the transitive verb ὑποτίθημι takes abstract nouns (perhaps *qua* concepts?) as its direct object. This difficulty – that Plato and Greek philosophers in general sometimes put nouns where we find a sentence more natural – is of course familiar from elsewhere. Consider a typical example from Aristotle, when describing Leucippus' position as a compromise between Eleatic philosophy and the appearances (*GC* 325 a 25–27: "For these things he agrees with the appearances, while <agreeing> with those who support the One (τοῖς δὲ τὸ ἓν κατασκευάζουσιν) that motion is impossible without void": the Eleatics are here designated by the synecdoche 'those who support the one' (rather than: 'those who believe that only one thing exists'). But it is clear, in this case, that the noun stands for what we may paraphrase as a proposition. Is this the case here, too with Plato's ὑποθέσεις?

To interpret the passage at hand, there are several alternatives. One is to imagine a verb, unlike our 'hypothesizing', which indeed takes concepts for objects. This is what Solmsen did.[5] According to Solmsen's interpretation,

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[4] B. Jowett, L. Campbell (edd.), *Plato's Republic* III (Oxford 1894) 309.

[5] F. Solmsen, *Die Entwicklung der aristotelischen Logik und Rhetorik* (Berlin 1929).

mathematicians used words such as 'odd' and 'even' without giving grounds to their usage, i. e., without defining the terms. This is a minimalist interpretation of what the mathematicians did, in a way, for, according to this interpretation, there was no special act involved in the ὑποθέσεις at all. The mathematicians simply went happily on, and this is what constituted the ὑποθέσεις.

Other interpretations typically assume that, at last partly, the verb ὑποτίθημι does not *really* take abstract nouns as its objects. It takes, besides those objects or instead of them, *propositions*, which have something to do with the noun in case.

Obviously, once we have freed ourselves from the text our imagination may run freely. Taylor, for instance, credited the mathematicians with postulates such as "all numbers are integers".[6] This is probably the maximalist interpretation for, according to this interpretation, the ὑποθέσεις certainly involve much thought and care.

It is easy to think of two less demanding interpretations, both still taking ὑποθέσεις to be propositional, while retaining the special relation with a certain noun. One is that ὑποθέσεις are definitions, and this is Mueller's view.[7] So an ὑπόθεσις would be something like "An odd number is that which is not divisible into two equal parts, or that which differs by an unit from an even number".

Another possible interpretation could be to take ὑποθέσεις as existence-assumptions. This was Ross's view,[8] and, according to this interpretation, an ὑπόθεσις is something like "Odd numbers exist".

We may therefore sum up the main available options as follows:

a. Mathematicians do not define terms (Solmsen).

b. Mathematicians define terms (Mueller).

c. Mathematicians postulate propositions of the form "X exists" (Ross).

d. Mathematicians postulate propositions of the form "X has property Y" (Taylor).

2. POSITIONING THE TEXT IN MATHEMATICAL CONTEXT

Each of the options reached above is a statement concerning Plato's impression of the mathematical practice of his, or of Socrates' time. The nature of the mathematical practice itself should therefore help us to dispose of

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[6] A. E. Taylor, *Plato, The Man and His Work* (London [4]1937) 291.

[7] I. Mueller, "On the Notion of a Mathematical Starting Point in Plato, Aristotle, and Euclid", in: A. C. Bowen (ed.), *Science and Philosophy in Classical Greece* (New York 1991) 83.

[8] W. D. Ross, *Plato's Theory of Ideas* (Oxford 1951) 51.

some of the options; and we may probably start by eliminating option (a). It is indeed vastly improbable that no definitions at all were available in Plato's time; I won't go into the evidence here, but simply refer to Mueller.[9]

The evidence for the mathematics of Plato's time is difficult for options (c) and (d) as well. Here indeed we are dealing with one equation having two variables. We know that Plato refers to a certain mathematical background, but we do not know exactly how, and we do not know what exactly that background was. However, some interpretations of the passage involve a fantastic level of mathematical achievement and should therefore, I think, be excluded. I am not alluding here at the moment to the unfounded suggestions of Taylor. Any interpretation following option (d), taking ὑποθέσεις as postulates, would be as problematic. For then the result would be that in Plato's time mathematics was widely known to be based upon a certain set of postulates which were unquestioned by the mathematicians, i.e. unanimously accepted. A minute's reflection would show that this would have been impossible at any time prior to the final canonization of the *Elements* in Late Antiquity. The postulates of Archimedes are unlike those of Euclid and, when he does introduce them, he may apologize for them or even argue for them: that is, postulates are not *ipso facto* taken for granted![10] If postulates were at all circulated *before* Euclid, they must have been as hotly contested as any other item on the intellectual agenda. The only alternative would be to say that the postulates in question are taken as agreed not in the absolute sense, that everyone takes them to be true, but in the local and relative sense, that they are not questioned in the context of the particular proof or treatise. This is not in contradiction to what we may assume for Greek mathematical practice; but the point now becomes subtle and non-obvious which, as I shall argue below, is better avoided in the interpretation of this passage. I thus find option (d) highly unlikely.

The same, *a fortiori*, holds for option (c) – that the mathematicians fail to argue for existence claims. This is in fact something Greek mathematicians hardly noticed at all. Knorr[11] has shown that the assumptions of existence are largely left uncharted in Greek mathematical practice: so, for instance, the assumption of the existence of the fourth proportional, given three line segments, is never even mentioned as calling for a special postu-

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[9] Mueller (n. 7).

[10] See the discussion of Archimedes' Axiom in, e. g., E. J. Dijksterhuis, *Archimedes* (Princeton 1987).

[11] W. R. Knorr, "Construction as Existence Proof in Ancient Geometry", *Ancient Philosophy* 3 (1983) 125–148.

late. One doubts that Plato noted this, and surely Glaucon would not have been able to understand the point.

The last remaining standard alternative, that ὑποθέσεις are definitions, is the only one plausible from the point of view of the mathematics of Plato's time, as shown by Mueller.[12] If the standard interpretation has any chance at all for survival, it is in this form.

So let us consider the emerging account: mathematicians start off with certain definitions; they do not give account of these definitions, and they are (merely?) consistent. Such is the outline of what may be considered perhaps, today, the established interpretation. Thus the mathematical practice is that of making certain definitions at the outset of... but then of what?

We should look more closely at this question. What is the situation being envisaged in the text? A certain practice is alluded to, and this practice is defined by verbs:

> Section 1: ὑποθέμενοι ... ὡς εἰδότες ... ποιησάμενοι ὑποθέσεις ... οὐδένα λόγον ἀξιοῦσι διδόναι ... ἀρχόμενοι διεξιόντες τελευτῶσιν ὁμολογουμένως ἐπὶ τοῦτο οὗ ἂν ἐπὶ σκέψιν ὁρμήσωσι.
> Section 2: προσχρῶνται ... ποιοῦνται ... διανοούμενοι ... ποιούμενοι ... γράφουσιν ... πλάττουσιν ... γράφουσιν ... χρώμενοι ... ζητοῦντες ... ἰδεῖν.

Such are the actions of the mathematicians, and the most striking thing is how *active* they are. The present of repeated action is the dominant tense (dominant in the sense of being the most frequent, and of being used in the main clauses). Mathematics is being evoked here, not in the way appropriate to the closed, static systems of concepts; rather we are meant to imagine people – many people – keeping doing things in a dynamic, open environment. They start off with an ὑπόθεσις, they go on through the rest of it, and then conclude – that is, conclude for as far as the matter at hand was concerned, and we are left to imagine them returning to do the same thing over and over again, which, in terms of the dramatic invocation, is what happens in between the two sections: they do a thing in section 1 and then they do some more in section 2. Plurality is the mark of the passage – which, after all, talks not merely of people engaged in 'geometry and calculation' but of people engaged in 'geometries and calculations' – περὶ τὰς γεωμετρίας τε καὶ λογισμούς. Plato makes clear that he refers to many, repeated acts.

It must be pointed out that the τοῦτο οὗ ἂν ἐπὶ σκέψιν ὁρμήσωσι can not be a *branch* of mathematics (with one being meant to imagine Eudoxus,

[12] Mueller (n. 7).

surrounded by a flock of disciples, heaving a sigh of relief, having finally written down the whole of his solid geometry). Branches do not end, and there is nothing to finish ὁμολογουμένως about them. By the end of a treatise, one reaches an understanding, perhaps. But agreement? As if you suspended belief through the entire book, but with the end of the final proposition you said against yourself "By god! He has convinced me now all right!". This is not a plausible way of thinking about reading *books* representing *disciplines*. A specific proof-event, taking place in a more oral exchange, seems to suit the description better.

As I will immediately argue, we should not imagine Plato here as describing some far-fetched, exotic procedure. He should be describing what was typical for mathematicians in the early part of the fourth century. Now, there is a great cloud of uncertainty surrounding the historicity of Proclus' account of the early history of Greek mathematics.[13] This account is framed, occasionally, in terms of book-production: Hippocrates of Chios, as well as Leon after him, have written books of *Elements*; Hermotimus of Colophon has written on *loci*. But from the pen of a late ancient author, writing an isagogic work, one would expect to hear much more by way of bibliography. (In fact, much of Proclus' account is framed in terms of chains of transmission, evoking not the trope of books but the trope of schools.[14]) I have argued [15] that we can see a transition occurring around Plato's time, changing mathematics in a more 'literate' direction: namely, letters were introduced into the geometrical diagram where, before, one would refer to the diagram through its qualitative features. As I have insisted there, there is no need to take on board any of the fanciful suggestions made by authors such as Havelock,[16] as if a sharp hiatus is noticeable in Greek culture at around

[13] The question I am alluding has especially to do with the authorship of books of *Elements*, which predated Euclid, according to some readings of Proclus (*In Eucl*. 66. 7 Friedlein). This text by Proclus has been much discussed, especially in and since W. Burkert, *Weisheit und Wissenschaft: Studien zu Pythagoras, Philolaos und Platon* (Nürnberg 1962); Engl. transl.: *Lore and Science in Ancient Pythagoreanism* (Cambridge, Mass. 1972); for a powerful restatement of the traditional view concerning Pythagoras, see L. Zhmud, *Wissenschaft, Philosophie und Religion im frühen Pythagoreismus* (Berlin 1997).

[14] For the isagogic genre in mathematics, see J. Mansfeld, *Prolegomena Mathematica from Apollonius of Perga to the Late Neoplatonists, with an Appendix on Pappus and the History of Platonism* (Leiden 1998).

[15] R. Netz, "Eudemus of Rhodes, Hippocrates of Chios and the Earliest Form of a Greek Mathematical Text", *Centaurus* (forthcoming).

[16] E. g.: E. A. Havelock, *The Literate Revolution in Greece and its Cultural Consequences* (Princeton 1982).

Plato's time from the 'oral' to the 'literate'. Indeed, following Gavrilov's final refutation of the myth of the absence of silent reading in antiquity[17] such categorical theses should best be avoided altogether. It is clear that already Hippocrates of Chios was known, perhaps primarily, as the author of a written text, and so the oral and the written were always present, simultaneously, in early Greek mathematics. Still, it is fair to say that the overall direction of change, from Classical to Hellenistic times, was in the direction of making mathematics more of a book-activity and less of a spoken activity. Certainly at Plato's time, let alone Socrates' time, there were only a few papyrus rolls available containing mathematical contents; while, at the same time, there were a number of active mathematicians with whom Plato was familiar. If only for this reason, then, it is likely that the form of mathematical presentation Plato would be most familiar with – the paradigmatic form of mathematics – would be that of the spoken presentation.

So, to begin with, we should not expect Plato to describe the mathematical situation as a relation between written elements in books, some of which are ὑποθέσεις, some are their results; rather we should look for a relation between lived and spoken acts. A relation between lived and spoken acts is indeed what Plato's tenses seem to imply, and the impression we should get, if we read the text without heeding any prior interpretation, is that the acts of setting up ὑποθέσεις are local to the acts of proof, just as the drawing of diagrams is. This is startling, if indeed ὑποθέσεις are hypotheses. Did Greek mathematicians, as a rule, begin their proofs by setting up the postulates they required for those proofs? If indeed the ὑποθέσεις are some general statements about mathematical objects, or if they are definitions of those objects, their mention should not be local to a local proof. The text, at this point, seems to imply a strange mathematical practice, one where all stories must start from the egg. Was this really the standard Greek mathematical practice? One immediate counter-example is the only text we are familiar with from before Plato's time – once again, Hippocrates' of Chios *Quadrature of Lunules*. In this case, it seems clear that *all* the 'axiomatic' preliminaries to the text are not original, but were introduced by a later redactor of Hippocrates (see Netz,[18] following e. g. Mueller[19]).

[17] A. Gavrilov, "Techniques of Reading in Classical Antiquity", *ClQ* 47 (1997) 56–76 (tr. with postscript by M. F. Burnyeat).

[18] Netz (n. 15).

[19] I. Mueller, "Aristotle and the Quadrature of the Circle", in: N. Kretzmann (ed.), *Infinity and Continuity in Ancient and Medieval Thought* (Ithaca – New York 1982) 146–164.

If so a written text, than *a fortiori* any spoken proof. The standard account, then, is just unlikely to have been the standard Greek mathematical practice.

But should we take normal Greek mathematical practice as our model here? I think we should.

The point is very simple. Section 1 is Socrates' second try. Just before that, he said to Glaucon, among other things (510 b 4–6, Cornford again):

> ...The mind uses as images those actual things which themselves had images in the visible world; and it is compelled to pursue its inquiry by starting from assumptions and travelling, not up to a principle, but down to a conclusion...

whereupon Glaucon replied, obviously justified (510 b 10):

> I don't quite understand what you mean.

And the text goes on, trying to elucidate Socrates' first try. 'Elucidate' is the key word. It just won't do for Socrates' second try to be inaccessible, and in some ways Socrates' second try clearly aims at accessibility. For instance, Socrates gives much more examples than are really necessary. He says 'geometry, arithmetic, etc.' rather than 'geometry, etc.'; 'odd and even, figures, three angles, etc.' rather than 'odd and even, etc.', etc. So Plato is at least setting up a show of accessibility.

That the text is not all that crystal clear is of course true – otherwise this paper would have been redundant! – but Glaucon, at least, was satisfied. He reacted to section 1 by (510 d 4):

> Yes, I know that.[20]

Irony could not, indeed should not be excluded. Probably Glaucon does not get quite to the depths of the matter. However, Glaucon understood *something*, recognized as known *something* – and it would have been poor taste to mislead him on purpose here. Glaucon's initial perplexity did not reflect any obtuseness on his part. Probably the good reader of Plato's time would have shared Glaucon's original perplexity and ensuing understanding. Glaucon, of course, is portrayed throughout the *Republic* as the clever brother. He is the best, short of Socrates himself. Socrates' section 1 should be such as to allow Glaucon a fair chance of getting his meaning. So when a reference is being made to something

[20] Cornford's rendering here has something of the understatement about it: the original πάνυ μὲν οὖν, ἔφη, τοῦτό γε οἶδα seems much stronger.

which Glaucon is supposed to know from first-hand experience, we should expect this minimum, that the reference is to what Glaucon actually knows from first-hand experience. Otherwise the "explanation" offered is a pointless joke.

I think that this consideration alone should make us rethink the established interpretation of the passage. It is very difficult to see how the statement "Mathematicians do not give grounds for their definitions" could have been obviously true from the point of view of Plato's good reader, let alone from Glaucon's point of view. It is even stranger to imagine the same with the statement "Mathematicians do not give grounds for their postulates". The modern educated reader has learnt some meta-mathematics, in progressive algebra classes, in a popular book of philosophy or in college logic classes; but I refuse to believe that the Classical Greek educated reader knew anything at all about meta-mathematics. However, he may have known something about mathematics, as opposed to meta-mathematics. He may not have known the logical structure of treatises, but he may have known the shape of proofs, for he may have attended some acts of proof. And here we return to the earlier point concerning the use of verbs in section 1. To be accessible to Glaucon as an obvious example, then, what is at stake in section 1 should be something that is involved in every separate act of proof, in the same way in which diagrams are.

3. THE RELATIONSHIP BETWEEN SECTIONS 1 AND 2

Diagrams, indeed, are the subject matter of section 2, and the relationship between the two sections merits a close inspection.

One feature which is shared by all the interpretations of the text I have come across is that, to some extent, they treat of the two sections as parallel.[21] Both sections – according to such interpretations – share the same structure: they (i) describe a state of affairs and then (ii) criticize it (criticisms that, according to some interpretations, are meant to transform mathematics, according to others – are meant to describe an inevitable feature of mathematics).

It is remarkable that the two paragraphs are actually not at all parallel, in fact they are unparallel to the point of stylistic unease. I quote again Cornford's translation of the beginning of section 2, 510 d 5–6:

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[21] Guthrie is typical: "First, they posit certain things... Secondly, they make use of visible models..." (W. K. C. Guthrie, *A History of Greek Philosophy* IV [Cambridge 1975] 509).

> You also know how they make use of visible figures and discourse about them, though what they really have in mind is the originals of which these figures are images...

This is in various ways unlike a literal rendering of the original. To offer a very lame one:

> Surely [you know] also that they make use of visible figures and discourse about them, not having those [i.e. visible figures] in mind, but those of which these [figures] are images.

The literal rendering is uneasy because it is a literal rendering – but also because of the thought it contains. This has three elements:

a. Mathematicians make use of diagrams.

b. Mathematicians do not have diagrams in mind.

c. Mathematicians have the modeled things in mind.

Cornford puts (a), or rather the entire (a)–(c), under a "how". But there's no "how" in the original. Glaucon was assumed to know "that", not "how". Secondly, Cornford's "though" is misplaced: he puts the concessive connector between (a) and (b)–(c), while in reality it is placed between (b) and (c). Finally and most importantly, Cornford does not render the way in which the verbs of (b) and (c) are participles dependent upon the finite verbs of (a). In sum: according to Cornford, Glaucon is meant to affirm that mathematicians have a certain practice, made up of two elements: (1) use of diagrams; (2) having the originals of diagrams in mind. According to Plato, however, Glaucon is meant to affirm that the mathematicians' practice of using diagrams (which is in itself assumed and therefore is not to be affirmed as such) involves having in mind originals, not diagrams. Cornford's version yields a certain duality ("diagrams – yes; but having originals in mind"), superficially resembling that of section 1 ("ὑποθέσεις – yes; but 'consistency' "). Plato's version – and here arises the unease for the modern reader – bears no resemblance at all to section 1.

This is a problem because of the καί at the beginning of section 2, which invites us to assume some continuity between the two. Anyway, without some such continuity the text loses much in stylistic terms – always a valid consideration in Platonic exegesis. How is the continuity to be understood, if not in terms of parallelism?

Let's note the discontinuity between the two sections. The claim of section 1 – that mathematicians use ὑποθέσεις – is not qualified; whereas the entire thrust of section 2 is to serve as qualification of the mathematicians' use of diagrams. Section 1 is devoted to A Bad Thing, reliance upon ὑποθέσεις as if they were known; section 2 is devoted to A Good Thing, having originals rather than diagrams in mind.[22]

Section 1 blames, section 2 praises. 1 asserts, 2 qualifies. The two should be somehow interconnected. It is supremely tempting to say that section 2 simply is a qualification of section 1. It is almost tempting to say that ὑποθέσεις are diagrams: which, I believe, is nearly the truth.

4. THE EVIDENCE OF DIALECTIC

I am about to offer my own interpretation. One of its merits, I believe, is that it makes the implications of the passage more credible, both for Plato's philosophy and for his contemporaries' mathematics. In order to make this more apparent, I would like to remark briefly on the problems of the standard interpretations from this point of view.

The most important contextual consideration, certainly from Plato's point of view, is that involving Plato's conception of dialectic. The whole of the Sun-Line-Cave set of analogies has as its goal the elucidation of the nature of dialectic (as well as the correlated "theory of Forms"). The *Divided Line*, at the end of which our text is situated, takes 'analogy' literally by using the formal and mathematical concept of 'analogy', namely that of proportion. The well-known formula is that (Reflection-mindedness):: (Objectmindedness)::(Mathematics):(Dialectics)::(Reflection-cum-Object-mindedness):(Mathematics-cum-dialectics). If three of the items are given, the fourth is given as well, so by understanding what reflections, objects and mathematics are, we can understand what dialectic is.

The problem is, of course, that there is no non-metaphorical way in which the relation between 'disciplines' such as mathematics and dialectic could be equivalent to the relation between looking at reflections and looking at objects. Whatever is said about dialectic which is non-metaphorical is due to what is directly said to distinguish it from mathematics. This means that whatever is our interpretation of the passage concerning mathematics, it should be imported, negatively, into our interpretation of Plato's dialectic. This is standardly done. However, as this involves im-

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[22] That section 2 ascribes 'A Good Thing' to mathematicians is all the more obvious when it is realized – as many would admit today, following Burnyeat (M. F. Burnyeat, "Platonism and Mathematics: A Prelude to Discussion", in: A. Graeser [ed.], *Mathematics and Metaphysics in Aristotle* [Bern 1987] 212–240) – that diagrams, according to Plato, are an inevitable part of mathematics. Hence, the *best* mathematicians can do is to have in mind originals. This they do, according to section 2: it is therefore a section of unalloyed praise.

porting the standard interpretation of our text, several results follow which are difficult.

One is that dialectic comes to be characterized as a system of propositions. Dialectic is "where all the propositions are proved" or "a system starting from definitions which are all argued for". Mueller, using expressions characteristic of this line of interpretation, says that "We might ... view the Platonic universal science as a two-tiered system with the following structure...".[23] This, I think, must be wide off the mark. Plato's dialectic was not a 'universal science', nor was it a 'system', nor had it any 'structure'. Plato's dialectic was a method, a way of doing things with words. Plato could hardly be more explicit on this matter: expressions such as ἡ διαλεκτικὴ μέθοδος are commonly used by him.[24] Dialectic is an *act*, not a *structure*. It will be seen that this is exactly parallel to the claim made above, independently, that the referent of section 1 must be a mathematical act rather than the structure of mathematics.

A further difficult result from the interpretation of dialectic as a propositional system is that it might involve Plato in the belief that everything can be proved. This results if we think that the criticism included in section 1 is to the effect that mathematicians fail to prove their first principles. Now the idea that everything can be proved was known in the Academy,[25] but Plato was deeply aware of the structure of ordered series, and there is no reason to suppose that he had any difficulties with such elementary logical observations such as those raised by Aristotle in the first chapters of *Posterior Analytics* – if anything, the *Theaetetus*' discussion of such structures is at a higher level of sophistication than that of the *Posterior Analytics*. It remains of course possible to argue (with Robinson) that the mistake ascribed to mathematicians by section 1 is not so much their failure to prove the ὑποθέσεις, but rather their very ignorance of their reliance upon ὑποθέσεις. According to Robinson's interpretation, then mathematicians are dogmatic where they should be agnostically hypothetical.[26] This would mean that the essential point made concerning dialectic and ὑποθέσεις is that dialectic, unlike mathematics, is fully aware of its use of ὑποθέσεις. Now in some ways it is clear that the difference between mathematics and dialectics is in the *way* in which the two use ὑποθέσεις: mathematics works forwards, dialectic backwards, etc. But there is another irreducible aspect of dialectic

[23] Mueller (n. 19) 83.
[24] See R. Robinson, *Plato's Earlier Dialectic* (Oxford ^{2}1953) 69 ff.
[25] See Aristotle, *Anal. Post.* I, 3.
[26] Robinson (n. 24) 152.

as distinct from mathematics, namely that it is in some sense *less* dependent upon ὑποθέσεις than mathematics is. Dialectic reaches an ἀνυπόθετον (Plat. *Resp.* 511 b 6); it is "destroying" the ὑποθέσεις (*Resp.* 533 c 8). It is true that in all this Plato is vague, but it is difficult to see how he can avoid the following dilemma. Either the criticism of mathematics involves the very use of ὑποθέσεις, or it involves the unreflective use of ὑποθέσεις. In the first case, the merit of dialectic would consist in its avoiding ὑποθέσεις – ridiculous! (i.e., ridiculous, as long as ὑποθέσεις are "hypotheses", for then the sense would be that everything is proved by dialectics). In the second case, the merit of dialectic would consist in its reflective use of ὑποθέσεις, which, while acceptable in itself, simply fails to characterize dialectic in anything like its projected Platonic shape – and, after all, would mean that *everything* is in principle like dialectic – all you have to do is to become aware of the "hypotheses" you use.

To make the interpretation of Plato's dialectic more credible, then, ὑποθέσεις should be the kind of thing that one could, in principle, make more or less use of; and they should be the kind of thing one makes use of, time and again, at the beginning of what one does.

5. HOW PROPOSITIONS BEGIN

I do not mean to suggest that Plato's use of ὑπόθεσις in the *Republic* is consistent (in the passages which most interest us, I suspect that Plato often wavers between the word ὑπόθεσις as referring to that kind of ὑπόθεσις which is used by mathematicians, and ὑπόθεσις as having a wider reference). But what is the general area of meaning within which the reference of the word may fluctuate?

Literally, an ὑπόθεσις is that which is put down. It is something set out and then left in one's immediate vicinity. It is not used much in concrete senses. Almost always, it is used with some abstract objects, usually those having to do with discussion and persuasion. The first translation offered by *LSJ* is 'proposal' and the similarity with the other main sense, 'supposition', as well as the generality of the word, are clear. Robinson suggests translating ὑποτίθεμαι as 'to posit as a preliminary'.[27] An ὑπόθεσις is, in the contexts which interest us, the preliminary part of a language-act, a preliminary part which may consist of a proposal or indeed an hypothesis. Besides being preliminary, some element of provisionality, of being non-final, imperfect, must be present as well. ὑπόθεσις is second rate: it is opposed to the better,

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[27] Robinson (n. 24) 95.

ancient method in *On Ancient Medicine* 1; it is the tool of the Second Sailing in the *Phaedo* (100 a 3) and, analogously, it is the route taken when the direct route has exhausted itself in the *Meno* (87 a 7). Of course, this sense of ὑπόθεσις being inferior is present in our own text.

Finally, the act of ὑπόθεσις is always verbal, and the ὑπόθεσις must be, on the whole, of a propositional nature, but it need not wholly be propositional, and it is easy to see how an ὑπόθεσις could take a noun as a direct object, if it says something or refers in some way to that object.

That the preliminary part of mathematical proofs of the Greek kind could be seen as such an ὑπόθεσις is, I think, obvious.

This preliminary part consists in drawing a diagram, while making some claims concerning its elements. These would be to the effect that such and such elements in the diagram *exemplify* (not "represent": I have argued,[28] that the most natural way to interpret Greek mathematical practice is that it behaved *as if* its object was just the diagram itself) certain geometrical or arithmetical objects. Most often, this preliminary stage would include an explicit assumption, e. g. that the object of inquiry is obtainable (this is known as the method of analysis)[29] or that a certain false proposition is true (which will lead to an absurdity; this is known as the method of *reductio*). Only following that preliminary stage will the deductive chain – the proof culminating in the result – begin in earnest. This deductive chain would move on smoothly, and when the result has been obtained the audience would have nothing left to do but to assent.

As far as Greek usage goes, then, the word ὑπόθεσις as used in section 1 of the text could *mean* nothing more precise than "the preliminary parts of language-acts", while *referring* to the specific preliminary parts of mathematical proofs, namely those involved in drawing a diagram, declaring it to exemplify the case in question, and making the preliminary assumptions concerning it.

Such an ὑπόθεσις takes nouns as its objects, namely those nouns which are mentioned while drawing the diagram. It may be helpful to notice immediately that this also agrees with the Greek mathematical usage itself as in, e. g., Euclid's *Elements* I. 26:[30]

ἡ ὑπὸ ΔΖΕ τῇ ὑπὸ ΒΓΑ ὑποκεῖται ἴση.

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[28] Netz (n. 15).

[29] Here it will be useful to remember Plato's early acquaintance with this form of argumentation, and his identifying this form as characteristically mathematical, for which see *Meno* 86 e 4 ff.

[30] J. L. Heiberg and E. S. Stamatis (ed.), *Euclidis Elementa* I (Leipzig 1972) 64. 4.

The [angle] under ΔZE was hypothesized equal to the [angle] under BΓA.

This refers back to a demand made right at the beginning of the diagram-drawing process:[31]

ἔστω δύο τρίγωνα ... ἔχοντα ... τὴν δὲ (sc. γωνίαν) ... ὑπὸ BΓA (sc. ἴσην) τῇ ὑπὸ EZΔ.

Let the [angle] under BΓA be equal to the [angle] under EZΔ.

This, indeed, is the only context in which the cognates of ὑπόθεσις are being used in extant Greek mathematical texts: namely, reference to acts being made in the preliminary parts of a proposition. Definitions and postulates, on the other hand, are never referred to in this way.

This interpretation fits the constraints on the interpretation we have developed above. It will be seen that the kind of ὑπόθεσις described here is a dynamic, repeated act, not a static, unique element in an abstract structure. It is also one that every educated reader with some experience of mathematics would immediately identify. And it is picked up nicely in section 2, which, while not discussing the ὑποθέσεις of mathematics in their entirety, does discuss the most conspicuous of them, and that which the Greeks associated most closely with mathematics – namely, diagrams.[32]

Dialectic could well employ ὑποθέσεις; but its ὑποθέσεις would not be as irreducible as the mathematical ones are. What mathematicians cannot hope to avoid is the statement "ABΓ is a circle" when it is not. This criticism would have been not only that of Plato, based on the theory of forms: it would be known to the historical Socrates and Glaucon, in the form of Protagoras' fragment (B7 DK) (embedded in Aristotle's statement of the criticism, *Metaph.* 997 b 35 – a 4):

Nor are the sensible lines such as the geometer says they are; for none of the sensible lines is straight or circular in this way <in which the geometer means it to be>; for the ruler does not touch the circle at a point but rather, as Protagoras said in his refutation of the geometers, <along a stretch of the circle>.

Now, why cannot the mathematicians simply avoid talking about sensible lines, in this way avoiding the Protagorean falsehood? The reason is

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[31] *Ibid.*, 62. 9–12.

[32] I argue in R. Netz, *The Shaping of Deduction in Greek Mathematics: a Study in Cognitive History*, Synthese Historical Library (Cambridge 1999) 35–43, that, generally, diagrams were the metonyms of mathematical activity in antiquity. This is proved, for the special claim that the word διάγραμμα simply meant 'proposition', in W. R. Knorr, *The Evolution of the Euclidean Elements* (Dordrecht 1975) 69–75.

clear: the mathematicians cannot avoid such claims because they can not, being mathematicians, avoid diagrams, and so avoid making those introductory make-believe statements where the diagram is taken to be the object itself. Thinking of the triangle-in-itself as much as they would, they are bound to start over and over again with their "let the triangle ABC be taken", while it is not. But dialectic is patently different for the simple reason it does not use diagrams. There is nothing obvious preventing dialectic from talking about fully abstract things, and so no false preliminaries are required. For any wrong preliminary, there is some hope to get rid of it in the process of dialectical argumentation; you may even hope to get correct preliminaries, eventually.

We may even say the following. The structure of argumentation in Greek mathematics is to make the hypothetical claim that P is – hypothetically – true; drawing the conclusion that Q is true as well; and then concluding from this the non-hypothetical, but conditional conclusion that from P Q follows. That this is the structure of Greek mathematical argument, I have argued in my book.[33] I have shown how the results of Greek mathematics do in fact take a conditional form, and I have argued that this represents the argument from the hypothetical assertions of the setting-out in particular terms to the conclusion derived in particular terms.

To us, it seems difficult even to imagine what a valid non-conditional conclusion is like: every argument is to us true only relative to certain assumptions. (Thus, famously, Russell has defined mathematics as a class of propositions all of the form 'P implies Q').[34] This of course already represents the realignment of the philosophy of mathematics following the discovery of non-Euclidean geometry. For Plato, there would be nothing impossible about the notion that a non-conditional conclusion could be rigorously proved: for instance, it could be rigorously proved that the Good is One. Not that, e. g. 'given the hypothesis of the forms, the Good is One' but rather 'the Good is One'. Of course, Plato would realize that any chain of argumentation must start at some point; but his reasonable assumption would be that one could prove that certain statements are true independently of the starting-point taken. One reasonable way to get such a result is by showing that the starting-point would be true, regardless of which possible assumptions are made. This could hardly be the case, however, in mathematics: what is irreducible in mathematics (but not in dialectic) is the starting-point that is fundamentally false, the one tied to the diagram. Any math-

[33] Netz (n. 32) ch. 6.
[34] B. Russell, *The Principles of Mathematics* (London 1900–1903).

ematical conclusion would depend not merely on a starting-point, but on a false one at that. In summary: mathematics is as good as a form of reasoning can get, with the proviso that it is tied to provisional false assumptions, which is the case because it is tied to the diagram: the images used define the science in question and the farthest one moves from images, the closer one gets to reality. This seems to be the main argument of the *Divided Line* passage and so the role I suggest for diagrams in Plato's characterization of mathematics is appropriate.

An argument against my interpretation is that it is simple. I make Plato refer to a well-known and concrete practice. How to account for his vagueness in this passage? It was a long stretch to have "the odd and the even" as shorthand for statements such as "the odd and the even exist". Is it not a longer stretch still to have it as shorthand for statements such as "let A and B, odd and even, be taken"? To the contrary, I would say that Plato's vagueness is an argument in favour of my interpretation. The practice I describe is concrete – but it is also complex. It includes the diagram, the statements which correlate the diagram with the proof, and the preliminary assumptions of the proof. Each of the three is an ὑπόθεσις in a different way: the first is not propositional at all, the second is false, the third is tentative. Glaucon would find it difficult to disentangle all of those components; but he would easily share the overall impression that mathematics is steeped with ὑποθέσεις, largely due to its reliance upon diagrams. The overall impression is correct, but vague, and this vagueness is reflected by Socrates' "odd and even". I suspect that the central image "odd and even" is meant to evoke is an expression of the form: "let A, an odd number, and B, an even number, be taken". I think it is clear that this is an ὑπόθεσις, and that it could be called "taking the odd and the even as ὑποθέσεις". (I return in the next section to mention another, more specific suggestion for an example Plato may have had in mind.)

The force of my account should be clarified. I do believe that the simple, literal sense of Plato's words – the one intended by Plato – is to describe the practice of mathematicians at the beginning of their propositions – and not, that is, the structure of mathematics taken as a single whole. At the same time, the following caveat should be made. The central model of the *Republic*, after all, is an assumption of isomorphism between the small scale and the large scale, the microcosm and the macrocosm; such an assumption has deep metaphysical motivation for Plato as, indeed, two things which share the same structure are, for him, as similar as any two things can be: this is what the theory of the Forms asserts. So, while Plato may be talking about individual propositions, he no doubt takes it as an immediate consequence that, whatever he finds, is thereby true of mathematics taken as a whole. Thus much of the standard interpretation of Plato's philosophy of mathematics can be salvaged even if we take the reference of ὑπόθεσις in the *Divided Line* passage to be, in itself, as narrow as I took it in this article. I have provided the main evidence for my reading above: the last, remaining section adds some further possible sources of evidence.

6. SOME POSSIBLE AVENUES

Having offered my interpretation, I now note three, more conjectural lines of argument that may perhaps be adduced in its favour, and then end by noting the possible philosophical significance of the account.

Very briefly, then, we should first of all note a textual curiousity. At the very beginning of section 2 mathematicians are said to use τοῖς ὁρωμένοις εἴδεσι, 'visible forms'. The variant reading of the Codex Malatestianus (plut. XXVIII. 4), however, has τοῖς εἰρημένοις εἴδεσι, 'the forms mentioned above'. That is, to the reader of the Malatestianus, Plato would explicitly say that while mathematicians use hypotheses, they use them for the sake of the square-in-itself. The interpretation offered in this article is the text of the Malatestianus.

The text of the *Republic*, based by Burnet (mostly) on four manuscripts, has few textual discrepancies, typically trivial omissions of particles and case-endings. Thus a substantial variant reading should be taken seriously. Could this be the right text? Or should we say that the εἰρημένοις, while a mistake, reflects an ancient interpretation of the passage, similar to that offered in my paper? Of course, neither has to be the case, and no special weight should be given to this last argument. Still, it is well worth our reflection: it would suffice for the textual history of Plato to go even a little differently (suppose *two* manuscripts had reported that reading...) and the interpretation of the *Divided Line* would have *had* to be different!

This, then, is one type of conjecture, having to do with Plato's text. I move on to another conjecture, having to do with Plato's literary methods.

Platonic writing on the whole is characterized by puns and clever plays of self-reflective irony. An interpretation of Plato's doctrine is supported, therefore, if it can show how features of the text may be taken as punning and metaphorical reflections of that meaning.

An obvious example comes right at the beginning of the passage I discuss. I refer to Socrates' words in Section 1, ῥᾷον γὰρ τούτων προειρημένων μαθήσῃ (510 c 1). Curiously, both translations fail with this simple phrase – perhaps because it is so much based on a pun. What Socrates literally says is

"You shall understand this more easily, after these preliminary things are said". There is not much point to Plato's word προειρημένων as opposed to the more simple εἰρημένων: Plato could well have said "You shall understand this more easily, after these things are said", but the 'preliminary' was added in, making the task so much more difficult for the translator. The only purpose of this phrase, I believe, is as ironic pun: putting Socrates in a position akin to that of the mathematicians, namely, he is explaining things with the aid of preliminaries. But this works, of course, only if we take ὑπόθεσις to mean a 'preliminary' and not something such a 'postulate'.

Indeed – going on to an even more speculative literary suggestion – the entire structure of the text may be taken as an ironic reflection of the structure of mathematical argument, moving from statement, through preliminary hypothesis and ensuing argument, to conclusion. Prior to our two sections, Socrates has stated the general claim in general, difficult-to-follow language – which we may compare to the general enunciation of a mathematical proposition. This is followed by a preliminary translation of the general claim to terms with which Glaucon is familiar, obtaining his agreement to the hypothetical claim. (And is not Glaucon's emphatic πάνυ μὲν οὖν, ἔφη, τοῦτό γε οἶδα [510 d 4] ironic, following upon Socrates' ὡς παντὶ φανερῶν [510 d 1]? Is Glaucon not too quick to assent rather than question the preliminary put forward?) Following further argument in section 2, finally, Socrates can return to the general statement that preceded this exchange, as if now a proved fact, in 511 a 3–8. Now this interpretation I offer is speculative on several counts: for one thing, we do not know that the mathematical propositions Plato was familiar with were at all preceded by general statements of the kind we are familiar with from Hellenistic mathematics. If they were, however – and there is no specific reason to believe they were not – then this interpretation, provided we take on board my overall account of what ὑπόθεσις means in this passage, results in a beautiful combination: section 1, the one discussing ὑπόθεσις, also functions in this metaphorical sense as an ὑπόθεσις itself – surely an ironic, reflective structure one would wish Plato to be credited with.

A final speculative possibility, suggested to me by Dmitri Panchenko, has to do with the mathematical example Plato may have had foremost in mind. It is an obvious feature of Plato's examples in this passage – odd and even, three types of angles, and square and diagonal – that they are not meant to allow any single proof event in which all three are taken together. Plato's aim is to suggest that his comments are true of all mathematical cases. However, an author typically has some examples in mind while working, and it is tempting to suggest that Plato may have in mind here the mathematical proof where odd and even, as well as square and diagonal, are essential – namely, the proof (as preserved in the transmitted text of Euclid's *Elements*) for the incommensurability of side and diagonal. Of course, the nature of proofs for incommensurability known to Plato – and Plato's knowledge about such proofs – are among the most vexed questions in the history of early Greek mathematics. (This, after all, is the theme of Knorr.[35]) Avoiding those questions, I just wish to mention a simple outcome: if indeed Plato has in mind the proof for the incommensurability of side and diagonal, than he has in mind *a single proof*. In the text preserved in the transmitted text of Euclid's *Elements*,[36] the hypothetical character of square and diagonal, as of odd and even, has a rich, complicated structure. The square and diagonal are mentioned as lines in the diagram, ΑΒΓΔ and ΓΑ (ΑΒ being the side; so this is ὑπόθεσις in one sense: set down in the diagram). The side and diagonal are hypothetically taken to be commensurable (ὑπόθεσις in another sense). If indeed commensurable, then they are as two numbers, here taken to be ΕΖ and Η (once again ὑπόθεσις in the sense of taking an object from the diagram to stand for an object – an especially glaring ὑπόθεσις in this case, as a line is taken to be a number). Then a complex set of ad hoc assumptions (ὑπόθεσις in yet another sense) shows that, if ΕΖ were to be odd, an impossibility would ensue, so that it is even, from which another impossibility follows. (And so the side and the diagonal are not commensurable.) If nothing else, this example may serve to show what I mean by my interpretation of ὑπόθεσις in the *Divided Line*. And the possibility that Plato has in mind this example is indeed tempting: after all, what else does come to mind just by mentioning 'square and diagonal'?

The above considerations – from textual criticism, from literary considerations, from the text of the *Elements* – are both highly speculative. I wish to conclude with what is a much stronger consequence of the account offered in this paper. I refer now to the overall nature of Greek mathematics as opposed to medieval and modern mathematics, and its possible consequence for the philosophy of Greek mathematics.

The overall divide between ancient and modern mathematics has been the subject of debate in the 20th century, especially in the context of the debate following Unguru.[37] The debate surrounded mostly the question whether historical differences are at all of much significance for mathematics: to the

35 Knorr (n. 32).

36 Eucl. *Elem.* III, 231–233 (see n. 30).

37 S. Unguru, "On the Need to Rewrite the History of Greek Mathematics", in: *Archive for History of Exact Sciences* 15 (1975) 67–114.

extent that history is allowed in, then it is usually agreed that the ancients differ in some fundamental way from the moderns. *How* they differ, however, is more difficult to say. The main proposal is that by Jacob Klein who argued[38] for a 'first-order' character of Greek mathematics as opposed to a 'second-order' character of modern mathematics. What this means in practice is a certain tying of the Greek mathematical proof to a local geometrical configuration, as opposed to the symbolic and more abstract approach typically taken by modern mathematics.

In a forthcoming book[39] I elaborate Klein's thesis as follows. It is not for nothing that early Greek mathematics strikes one as focused on the local geometrical configuration: it can be shown that ancient Greek mathematicians took pains to make sure their solutions to problems would differ from other solutions offered in the past. The aim was precisely that of locality: a solution was a solution for just *that* local problem. Instead of contributing to mathematics as a structure, adding on to the work produced by other mathematicians, the Greek mathematician aimed at his own individual glory to the *exclusion* of mathematical continuity. The Greek mathematical work aims to be a world to itself, wiping the slate clean from previous achievement and making itself inaccessible and singular. This is a mathematics where the aim is to endow the individual contribution with its own, special aura. We may easily see the context of this approach in the social setting of early Greek intellectual life, with the limited institutional setting and the focus on radical debate.

On the other hand, as we move on to Late Antiquity and the Middle Ages, a new approach seems to take hold. The aim of intellectual work is the re-arrangement and perfection of past achievements. The question of the position of a given contribution in the wider scheme comes to the fore. Instead of individuality, the goal becomes that of all-encompassing treatment. Thus problems are systematized and come to be seen as special cases of more general problems, fitting a larger classification. The local configuration loses its importance, and the focus is now on the overall scheme of possible problems and solutions.

In short, there is a historical process whereby mathematics becomes a system – which it already is the medieval Arabic world and most certainly is in modern Europe. This image of mathematics as a single, monolith system informs, still, our own image of what mathematics is. Instead of a philosophical

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[38] J. Klein, *Greek Mathematical Thought and the Origins of Algebra*, tr. by E. Brann (Cambridge, Mass. 1968).

[39] R. Netz, *The Transformation of Early Mediterranean Mathematics: from Problems to Equations* (Cambridge, forthcoming).

constant, this image is, I would argue, a historical construct. There is a specific historical transformation – from the classical Greek world into Late Antiquity and the Middle Ages – transforming mathematics from local configurations to overall schemes. Put succinctly: this is the transformation from mathematics with a plural *s* to Mathematics with a capital M. We would say that Mathematics *is*; but the Greeks would say that the *mathematika* – the (many diverse) things having to with the advanced studies – *are*.

If this is true for the difference between ancient and modern mathematics then we should also expect it to be a difference between ancient and modern philosophies of mathematics: that ancient philosophy of mathematics was rather more a study of individual acts of mathematics, while modern philosophy of mathematics is rather more a study of mathematics taken as a monolithic whole. The interpretative question concerning ὑπόθεσις in the *Divided Line* is primarily that – does Plato study mathematics as a monolithic whole (as the standard interpretations have it), or does he study individual acts of mathematical proof (as I suggest)? The main significance of the interpretation offered in this article, then, is to suggest how a philosophy of mathematics with the individual mathematical act as its focus would look like – in other words, what is the philosophy of mathematics with plurals.

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Статья посвящена вопросу о значении слова ὑπόθεσις в пассаже из “Государства” Платона (510 с – 511 а). Рассматриваются прежде всего место этого рассуждения в контексте ранней греческой математики и его внутренняя логическая структура. Маловероятно, что здесь подразумевается ссылка на какие-либо сложные метаматематические качества греческой математики, взятой как целое (например, что вся она зиждется на ряде не нуждающихся в обосновании принципов): едва ли такие соображения должны быть очевидны Главкону, как и нет уверенности в том, что воззрения греческих математиков в эту эпоху были столь однородны. Далее, автор стремится показать, что две основных части пассажа более тесно связаны между собой, чем это обычно выглядит в переводах, и соответственно весьма тесной выступает связь между обращением математиков к “гипотезам” и их обращением к чертежам. Наконец, высказывается предположение, что в данном специфическом контексте ὑπόθεσις указывает на предшествующий рассмотрению каждой теоремы акт начертания определенной фигуры, сопровождаемый формулировкой относящихся к этой фигуре предварительных утверждений.