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# Thibault and Science I.
# Measure, Distances and Proportions in the Circle

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**Abstract – In this paper we investigate the basic mathematical and philosophical tool of Gérard Thibault d'Anvers, the Circle. One of our main goals was to describe the Circle with coordinate geometry, and to estimate the rate of accuracy of his work. Furthermore, we also wanted to test the statements made by Thibault in his fencing manual, *Academy of the Sword* [Thibault, 1630; Greer, 2005]. To do this, we compared his observations and calculations with the results of available modern day and historical anthropometrical data sets. Based on our results, we also want to give some practical information about Thibault system for the fencers who study his art in our time.**

**Keywords: fencing, mathematics, anthropometrics, Thibault**

## I. INTRODUCTION

Gérard Thibault d'Anvers, the renowned Dutch fencing master used a unique approach when he described his fencing system based on the Spanish school of swordsmanship, La Verdadera Destreza. Thibault defined this system in his magnum opus, the fencing manual *Academie de l'Espee* or *Academy of the Sword*. Written in 1628 and first printed in 1630 [Thibault, 1630], *Academie de l'Espee* is considered one of the most detailed and exquisite fencing manuals and manually-printed books in history. During our research we used the English translation and analysis written by John Michael Greer [Greer, 2005].

The core concept of Thibault's fencing system is based heavily on strict mathematical and geometrical principles, with the use of the so called "mysterious circle" as the foundation of his work [Thibault, 1630]. Moreover, Thibault's teachings place great emphasis on the proportions and anatomy of the fencer, deriving the swords and its various accessories' measurements from the height of the fencer and the distance between several distinct points on the human body. These points are specified by the afore-mentioned "mysterious circle" and its properties, which when extended draw out several specific lines around and inside the circle when extended.

Thibault, as he himself describes in the second chapter of his manual, also compared his anatomical calculations to the extensive proportional studies of the famous German painter, mathematician and humanist Albrecht Dürer [Greer, 2005; Dürer, 1528]. The main reason for Thibault to include this comparison is to verify his claims for the reader about the rightness of his theory, but it is interesting that he also corrects Dürer in several points in the same chapter.

The initial goal of the authors was to determine whether different historical martial art systems can be modeled with analytic methods, and if it is possible, to define analytic models describing basic procedures and situations in fencing. For this we need a verified, precise and detailed fencing system. We chose the work of Thibault as our first target in this endeavor, because of the highly mathematical and geometrical nature of his fencing style, which makes it easier to approach the subject from a scientific standpoint.

This article is the first in a series of articles. Here we describe our findings on the subject of precision and accuracy in Thibault's system, and based on this, we also determine whether the system is suitable for our goals. In addition, we tried to outline the basic mathematical elements of Thibault fencing style, and build our models on the resulting principles. These models can be used as tools for those who would like to examine the work of Thibault from a more number based or scientific standpoint.

Finally, we provided some measurement values Thibault did not evaluate explicitly in his book (for example some measurements of the sword, or some measurements of the body) to give some practically applicable results in our paper for the fencers who want to understand and use Thibault's system as precisely as possible.

In the first step of this inquiry, we looked into the distances and the proportional transformations which form the basis of the Dutch master's teachings, and which are described in detail in the first two chapters of *Academy of the Sword*. Moreover, we also introduced a new measurement unit when dealing with Thibault's proportional work called Thibault Unit or ThU for short.

Since we use Thibault's system as a tool for modeling, and we chose the scientific standpoint, we will not be inquisitive about the philosophical background of his teachings. The philosophy behind Thibault's system is very interesting, and many of its basic ideas are still a mystery to us. However we are not experts in the required fields for a philosophical discussion about these questions, and the investigation of this topic has very different methods. As a consequence of our choice, from now on we do not use the notion "mysterious circle". We will call it simply the Circle, which means the circle itself and all the lines and marked points in and around it. To avoid ambiguities, we refer to the circle as geometrical category when it is written lowercase letters.

Regarding the other technical terms, we follow the translation of John Michael Greer [Greer, 2005].

The first thing we wanted to study when we started to view Thibault's manual from a scientific standpoint was the Circle. This diagram is the basis that Thibault built upon the various measurements of the sword and its accessories, the right fencing distances and the various steps and movements amongst many others. As he himself says in his manual: "For just as it is important to the commander of an army, in order to lay siege to some place, to know the most secure approaches, the passes, the nearby heights, the bodies of water, and in sum, all the situations around it; in the same way, the delineation of the circle is important to us in this exercise, for it is like a map of all movements which can be made and changed according to the variety of occasions." [Greer, 2005: 39]. Indeed, Thibault's Circle is a geometrically and proportionally constructed "map" which is individually tailored for the fencer, detailing the possible movements (including the proper distances, postures, weights and angles) one can make in different situations.

The main attributes of the Thibaultian Circle come from the human body itself (Circle No.1 of Tabula I). The center of the circle is the navel, if the person in question stands still with straight legs and with both feet on the ground. The diameter of the circle is from the base of the sole to the end of the fingers, if a person extends his arm straight upwards with the elbows touching the top of the head, which also means that the radius is measured from the navel to the sole or to the top of the fingers. This way the basic properties of the Circle are set. He divides the length of the diameter into 24 equal parts, and with this he defines a relative length measure, which we call Thibault Unit, or **ThU**.

Thibault calculates his results in a decimal system, and he evaluates the length of different distances up to 0.01 ThU. We would like to be a little more precise than this if it is possible, so we will calculate up to 0.001 ThU. This may cause some rounding errors, but these are trivial.

Thibault's system seem to be very precise, his limit of accuracy (0.01 ThU) is near 1 mm. But he based the geometry of the Circle on the proportions of human body as he refers to this many times in his book. To determine the real accuracy of his system, we compared his work with several anthropometrical sources throughout history, staring from Da Vinci's Vitruvian man to a modern anthropometrical study of our time. We did this to determine how close Thibault's derived proportions are to real life human proportions. We also looked at the statements he wrote about the proportional measures made by Albrecht Dürer.

Besides the laborious theoretical viewpoint, we give practical results based on the Circle and Thibault's descriptions to inquisitive fencers. We completed and verified his calculations about the Three Instances, we calculated the length of the paces Thibault uses in his system, and we determined the proper measurements of the blade in Thibault's opinion.

Our work consists of many mathematical calculations, most of them are based on coordinate geometry [Bronstein and Semendjajev, 1973]. But we do not want to bother every reader with the details of these calculations, not even the most important formulae of our results. In the body of this article we collected the results which can be described without complicated expressions, and which can be interesting for even those who are not interested in the mathematics of the Circle. Hence we collected every mathematical detail in the Appendices.

## II. THE CIRCLE

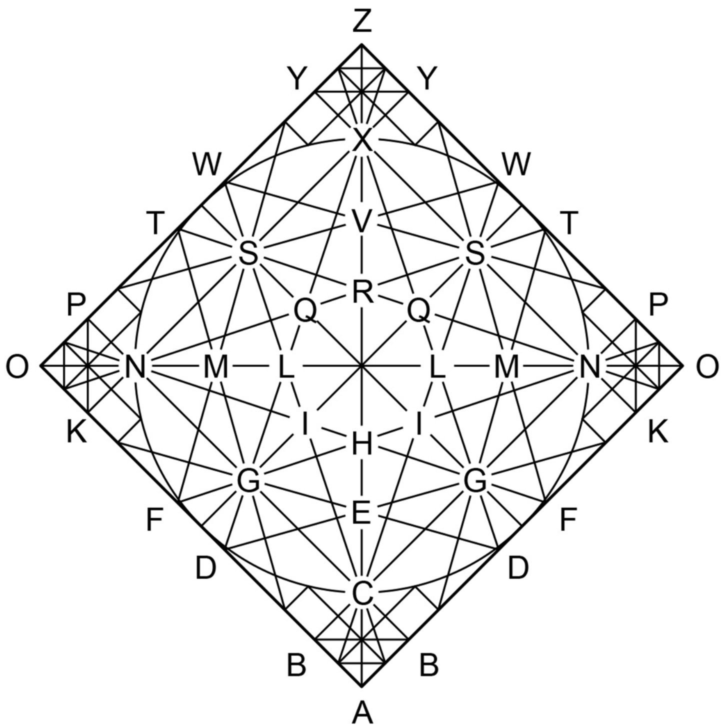
### 1. Drawing the Circle

One of our main goals was to describe the Circle in a coordinate system, as a prior base for modeling the fencing methods of Thibault. To do this, we followed his instructions in the drawing of the Circle in essence; however, we had to alter some steps during the evaluations to adjust for the logic of coordinate geometry.

Thibault splits the diameter of the Circle (which is defined by the height of the human body with fully extended arm held upwards) into 24 equal segments. With this he defines a relative length measure. From now on we use it as a Unit for our calculations, which we described as Thibault Unit (ThU). Furthermore, he divides this unit into 10 parts, and each of these parts into 10 minutes. This also determines the maximal accuracy of his system.

Since the measure of 1 ThU depends on the height of the fencer, it has no standard length value. In the case of a specific fencer it can be determined in some ways, which will be detailed later. In order to picture the meaning of 1 ThU we give two examples; the detailed evaluations will be described later. In the case of a person, who is 196.9cm tall (from sole to the top of the head), 1 ThU is equal to 10cm. Henceforth 1 minute is equal to 1mm. If the fencer is 175cm tall, 1 ThU=8.89cm, and 1 minute is equal to 0.889mm. We will give the length in centimeters, using the former two examples throughout the paper. ThUs we have to emphasize that 1 ThU is not equal to these values in general.

To calculate the coordinates of the points and to give expressions for the lines, first we have to define our coordinate system. We introduce a simple Cartesian coordinate system, where the pole is the centre of the circle, the 'x' axis points to the point $O_r$, with the vertex of the circumscribed square on the right side of the picture below. The 'y' axis points to Z.

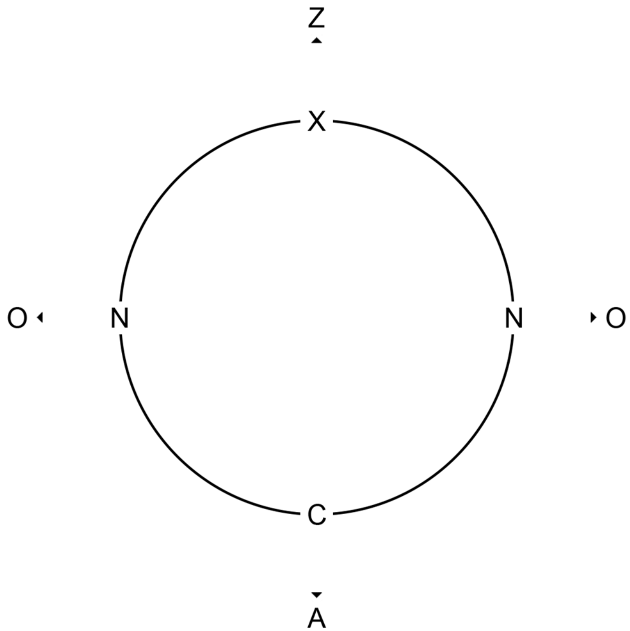



We show only the main logic of our evaluation, following the steps of the drawing of the Circle, and determining the most important points and lines in it. To do this, we show the tracing of the Circle stepwise, and name the points and/or lines to be described during the process. We tried to avoid giving numerous equations and coordinates here, so all our calculations can be found in detail in the Appendix for those who are interested in the mathematics of it, or would like to use it in education, training or research.

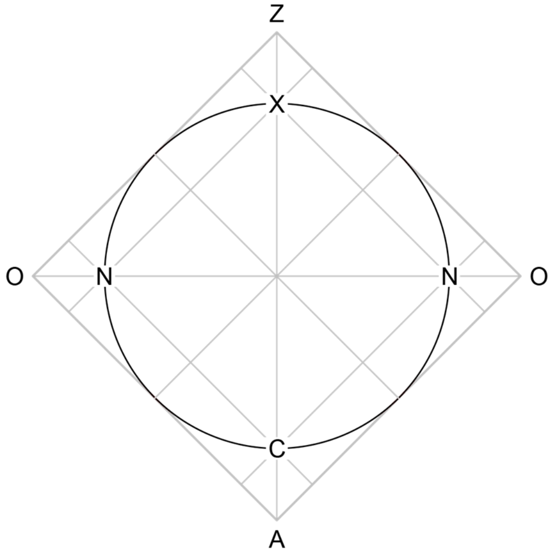
Though this way we can see the drawing process of the Circle in full, we still need to define the following notations. We mark these points with big letters following the notations of Thibault himself. If there are two points denoted with the same letter, we distinguish them with subscripts. Subscript 'l' denotes the points on the left half (with negative x coordinate value), and 'r' denotes the point on the right side (with positive x coordinate value). Furthermore, when it is necessary we will denote the lines with the letters of the defining points with a small line over them. For example $\overline{O_l O_r}$ denotes the line defined by the two O points.

The first step is to describe the circle, and the points determining the circumscribed square (A, $O_r$, Z and $O_l$), and the inscribed square (C, $N_r$, X and $N_l$). Furthermore, we introduce four auxiliary points ($É_{++}$, $É_{-+}$, $É_{--}$, and $É_{+-}$), which are the intersections of the circle and the lines of the circumscribed square. The lower indexes show in which quarter of the coordinate system the point is located. These evaluations are based on simple geometric and symmetry considerations.

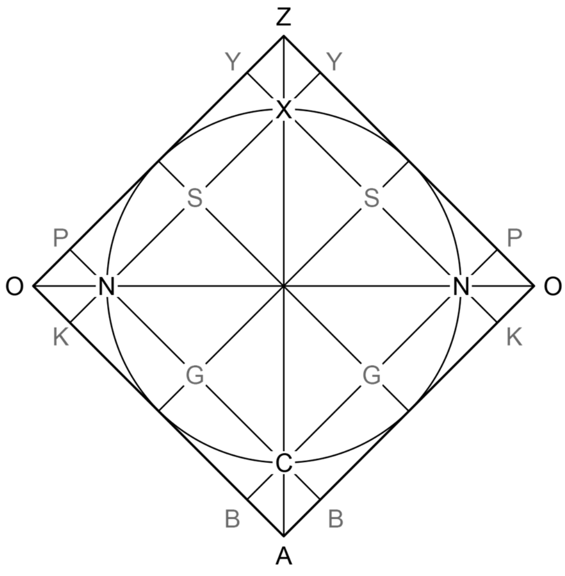
Note: since these auxiliary points are not part of the original Circle, the following pictures do not contain them.



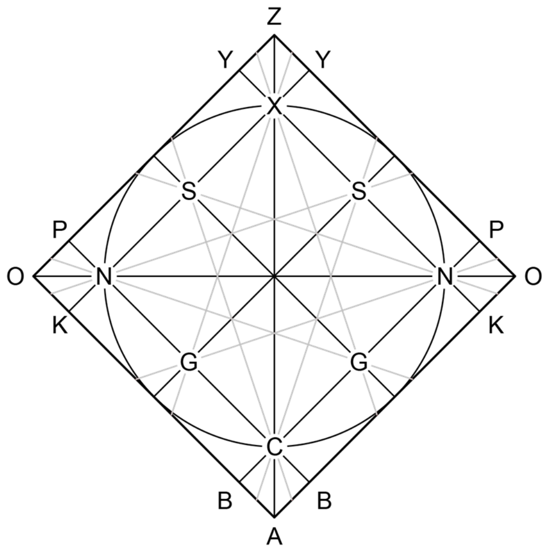
The second step is to draw the squares and give the equations for the sides, namely the two diameters $\overline{CX}$ and $\overline{O_l O_r}$ . Furthermore, we give the equations of the medians of the circumscribed square.



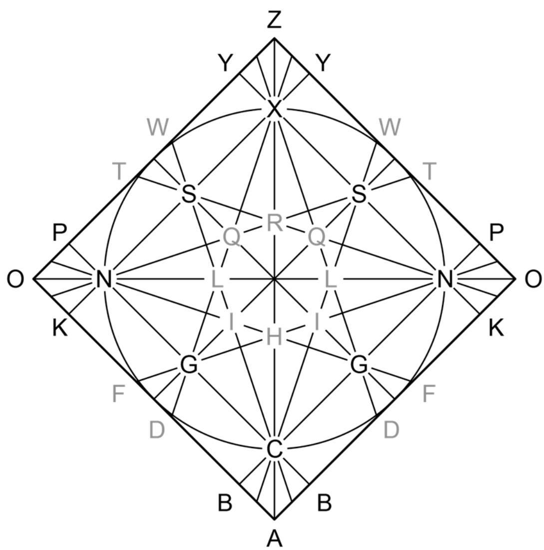

These lines will determine important intersections. Next we collect these intersections, and give their coordinate values, namely the points $G_l$, $G_r$, $S_l$, $S_r$, and $B_l$, $B_r$, $K_l$, $K_r$, $P_l$, $P_r$, $Y_l$, $Y_r$.



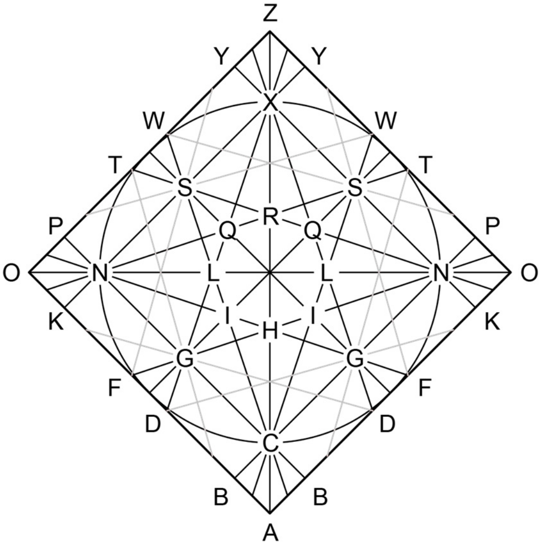
These points define the following lines. We calculate the equations of the lines drawn through Ss and C, Gs and X, and the lines drawn through Ss and Ns on the far side, and similarly for Gs and Ns. They are called inner collaterals and inner traverses (though Thibault defines them with other points we have not introduced yet).



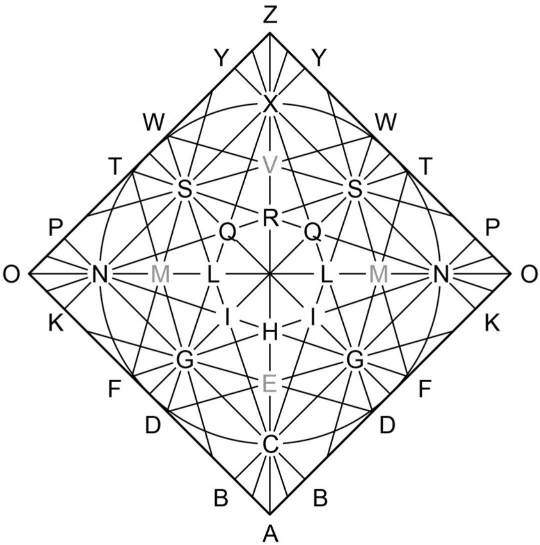

After the previous step many new intersections appear on these lines, but we do not need to determine them all. First we need some inner intersections, namely H, $I_l$, $I_r$, $L_l$, $L_r$, $Q_l$, $Q_r$ and R. Secondly, we need the outer intersections $D_l$, $D_r$, $F_l$, $F_r$, $T_l$, $T_r$, and $W_l$, $W_r$.



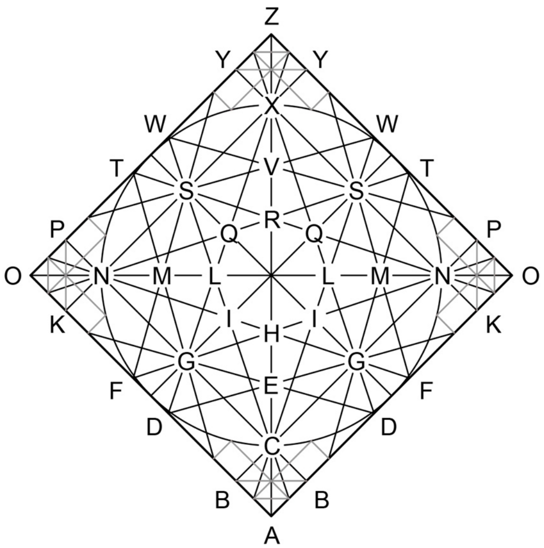
The last main lines are defined by the points mentioned above. These lines are drawn through Gs and the outer intersections Ds (on the far side) and Ts (on the same side), and symmetrically through Ss and the outer intersections Ws (on the far side) and Fs (on the same side). These are called outer collaterals and outer traverses.

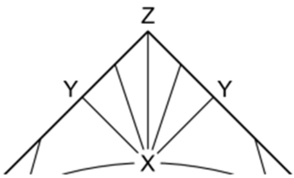


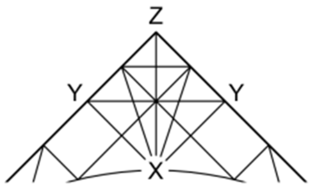

The Circle is almost complete, we only need the four missing inner intersections E, $M_l$, $M_r$ and V.



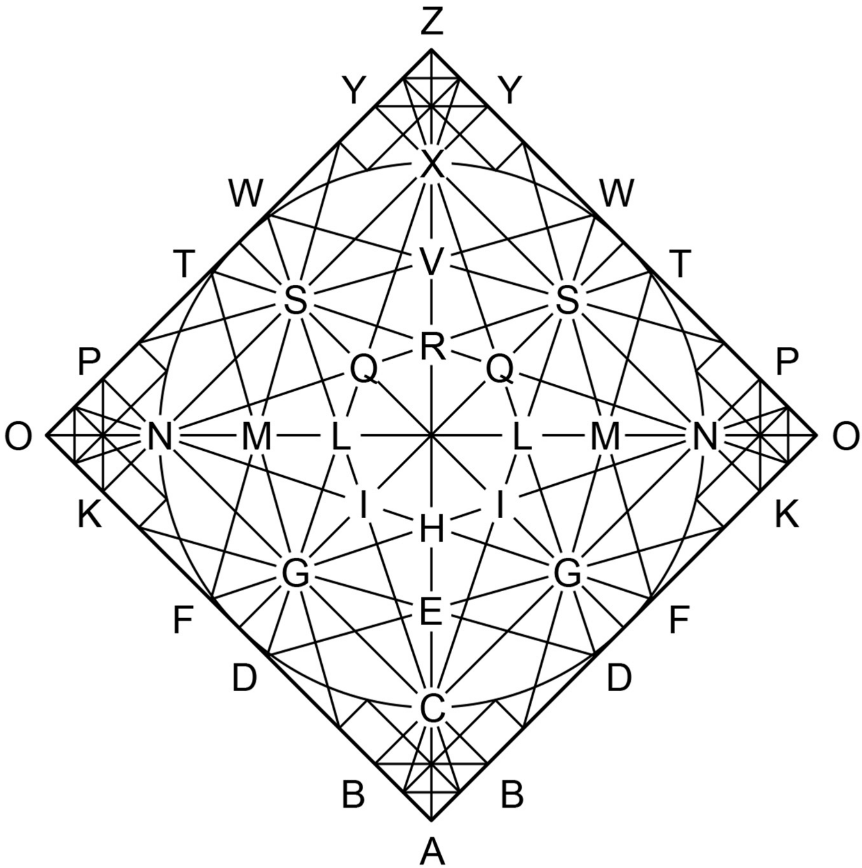
There remains one more additional step. We have to describe the short lines of the quadrangles (the small outer squares ABCB, XYZY, and two KOPN). To do this, we must first describe the medians and the missing diagonal of these quadrangles and – most importantly – the so-called “foot line” or “pedal”.



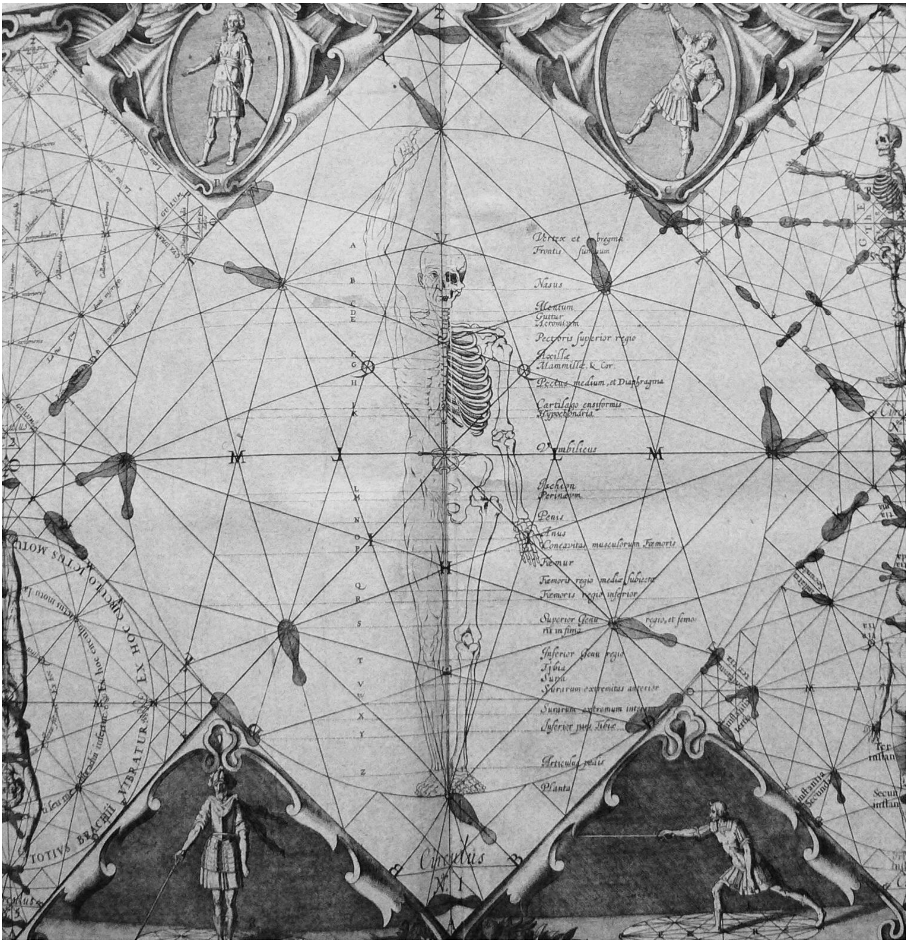
And now we have the full Circle here.




## 2. Proportions of the human body

Since one of the basic ideas behind Thibault's system is that every movement, measure and weight can be determined by the properties of the human body, we have to evaluate his statements. To do this first we need to determine the lengths of the human body parts he describes. Later we will give a comparison between his system and some results from modern anthropometry.

We recall here Circle No. 1 from the first Tabula. It shows the human body drawn on the Circle. Thibault designates several main points on the body, and shows their connection with the points and lines of the Circle. Some of these are trivial (point C, point E, or point V), and the picture shows indications for the others. For example he draws the small line denoted by B between the two S points.




*Circle No.1 of Tabula I*

The Appendix contains the details of our evaluations; here we only show the most important results. Although our expressions are derived in the coordinate system defined in the previous chapter, here we give every distance between the soles of the feet and the actual point on the human body.

| The member of the body | The defining geometry | Distance from sole | Distance from sole (cm) | |
|---|---|---|---|---|
| | | | (ThU) | (for 175/196,9 cm tall fencer) |
| Endpoint of finger (arm held straight above) | point X | 24 | 213.4 | 240 |
| Top of head | point V | 19.69 | 175 | 196.9 |
| Top of the face | line A | 19.243 | 171.1 | 192.4 |
| End of nose | line B | 18 | 160 | 180 |
| Chin | line C | 17.119 | 152.2 | 171.2 |
| Adam's apple | line D | 16.8 | 149.4 | 168 |
| Top of the shoulders | line E | 16.566 | 147.3 | 165.7 |
| Top of the chest | point R | 16 | 142.2 | 160 |
| Armpits | line F | 15.364 | 136.6 | 153.6 |
| Nipples | line G | 15 | 133.4 | 150 |
| Middle of the chest | line H | 14.4 | 128 | 144 |
| Breastbone | line I | 13.586 | 120.8 | 135.9 |
| Floating ribs and diaphragm | line K | 13.314 | 118.4 | 133.1 |
| Navel | centre of circle | 12 | 106.7 | 120 |
| Top of the hipbone | line L | 10.686 | 95 | 106.9 |
| Perineum | line M | 10.414 | 92.6 | 104.1 |
| Penis | line N | 9.6 | 85.3 | 96 |
| Anus | line O | 9 | 80 | 90 |
| Top of the thigh | line P | 8.636 | 76.8 | 86.4 |
| Greatest thickness of the thigh | point H | 8 | 71.1 | 80 |
| End of fingers of hanging arm (not in the text, referred later) | point H | 8 | 71.1 | 80 |
| Hollow of the thigh | line Q | 7.2 | 64 | 72 |
| Bottom of the thigh | line R | 6.881 | 61.2 | 68.8 |
| Top of the knee | line S | 6 | 53.3 | 60 |
| Bottom of the knee | line T | 4.757 | 42.3 | 47.6 |
| Top of the shin | point E | 4.31 | 38.3 | 43.1 |
| Top of the thick part of the calf on the inside | line V | 3.94 | 35 | 39.4 |
| Top of the thick part of the calf on the outside | line W | 3.364 | 29.9 | 33.6 |
| Bottom of the thick part of the calf on the inside | line X | 2.87 | 25.5 | 28.7 |
| Bottom of the shin | line Y | 2.272 | 20.2 | 22.7 |
| Ankle | line Z | 0.89 | 7.9 | 8.9 |
| Soles of the feet | point C | 0 | 0 | 0 |

*Table I. Distance of the typical points of the human body from the soles of the feet in ThU*

Based on these results, Thibault declares some of the proportions, which are amazingly accurate in his system. For example, it is trivial that the length of the arm together with the hand is the length of RH or equally HC, one third of the diameter, namely 8 ThU (71.1 – 80 cm).

But as another example, if we compare the length of the foot line and the distance between the top of the head and the chin we find some inaccuracy. The former is 2.485 ThU (22.1 – 24.9 cm), but the latter is 2.571 ThU (22.9 – 25.7 cm), which means that the difference is greater than the accuracy limit of Thibault, namely 0.086 ThU (0.76 – 0.86 cm). It is worth noting that this difference is smaller than 1 cm.

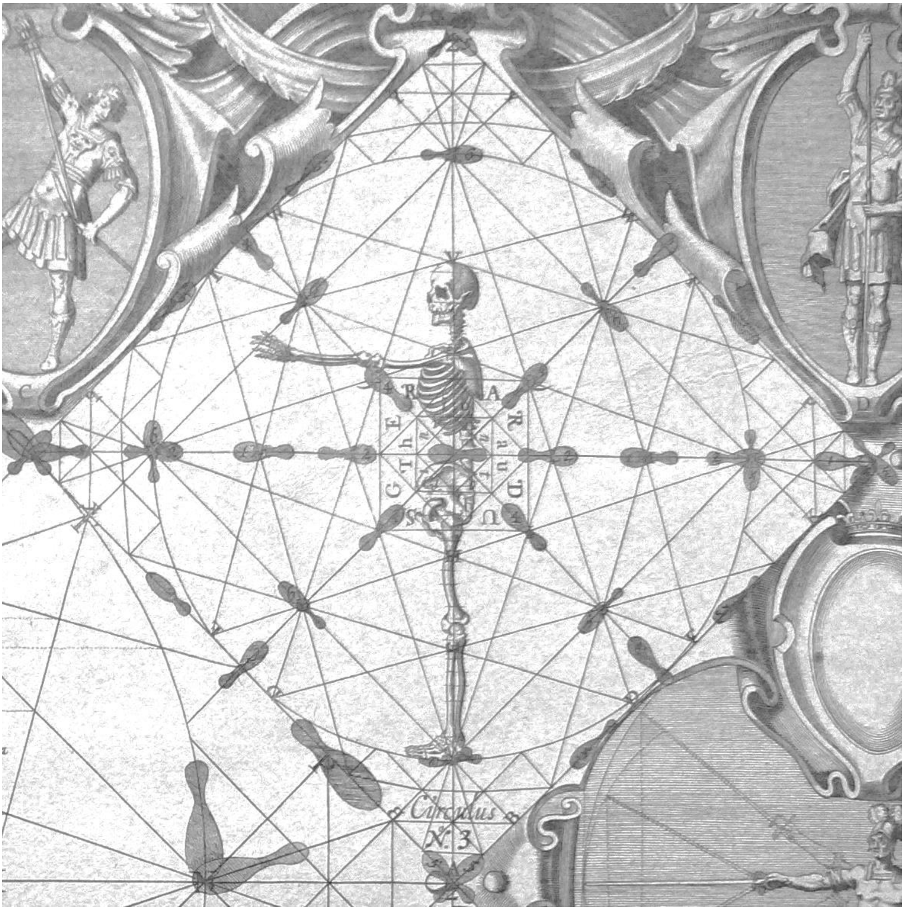
The other referred proportions cannot be evaluated by the Circle. One must investigate the details of the picture itself. For instance he refers to the width of the feet, but does not define it precisely in the Circle.

The above results will give us the opportunity to provide a short comparison between Thibault's system and the results of modern anthropometry. With this we can estimate the accuracy of the whole system based on the Circle. On the other hand, one can use this comparison to adapt the Circle to the fencers of the modern age.

## III. APPLICATIONS

### 1. Length of the steps

After the definition of the elements of the Circle (which is completed with the description of the proportions of the body), Thibault begins to use them to build his fencing system. First, he relates the lengths of the natural paces with the measures of the Circle. This is determined by Circle No.3 of Tabula I, where he shows the natural paces in six different situations (which can be simplified to three, if we interpret them mathematically), denoted by numbers. Now we investigate the statements of Thibault on this topic, and calculate the length of these paces.




*Circle No.3 of Tabula I*

The feet, denoted by ‘1’, show that the distance between $O_r$ and $O_l$ is equal to four ordinary paces. This distance is 33.941 ThU (301.7 – 339.4 cm). But one can see that the first sole touches $O_r$ with the heel, and the last one touches $O_l$ with the tiptoe. Since these steps are linear, we have to correct the above distance with the length of the sole (2.485 ThU).

Therefore the correct distance of four ordinary paces is 31.456 ThU (297.6 – 314.6 cm), and hence the length of one ordinary step is 7.864 ThU (69.9 – 78.6 cm). Using this, one can derive the exact coordinates of the heels (or, equivalently the tiptoes) in the Circle, which is shown in Appendix C. If one compares these results with the picture, he can find it very precise.

The feet denoted by ‘2’, ‘3’, ‘4’ and ‘5’ illustrates the same statement, namely that the diameter of the circle is equal to the length of three ordinary paces. With the use of the above result it can be calculated that the length of three paces is 23.592 ThU (209.7 – 235.9 cm) from heel to heel, or with the foot line correction 26.077 ThU (231.8 – 260.8 cm). The diameter of the circle is 24 ThU (213.36 – 240 cm), and this difference is accurately taken into account on the picture. Only the steps of feet '4' are seem to be a little smaller, but the difference is not noteworthy.

It is much more interesting to investigate the feet denoted by '6', since they show a different path, which will be important for Thibault, when stepping circularly. Although the centers of the soles are moving along the sides of the inscribed square, the movement is a bit circular, which can be seen by the directions of the soles on the picture.

First we have determined the length of the steps shown by these feet, and the references were the middle points of the soles. In this case the length of two steps is equal to the length of the side of the inscribed square, namely 16.97 ThU (150.9 – 169.7 cm). Hence one step is 8.485 ThU (75.4 – 84.9 cm) long, which is longer than the paces described above. The difference is 0.621 ThU (5.5 – 6.2 cm).

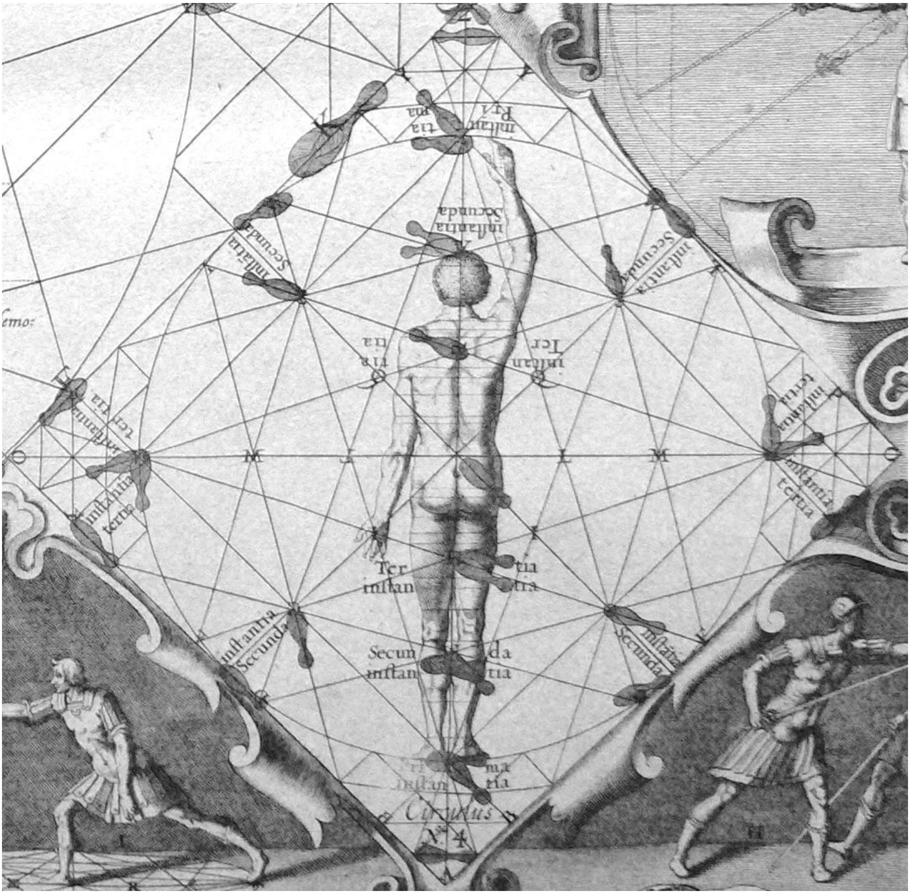
On the other hand, if one follows the above method but uses the endpoints of the heel as a reference, the length of one step becomes 8.169 ThU (72.6 – 81.7 cm). The difference in accordance with the former results, reduces to 0.303 ThU (2.7 – 3 cm). The details of the calculation can be found in Appendix C. If we take into consideration the dynamics of a step in Thibault's art, one can find the latter result more relevant.

## 2. The Three Instances

Thibault gives the most accurate evaluations when he discusses the three Instances, namely the First Instance, the Second Instance and the Third Instance. While describing the details of Circle No.4, he gives completely correct results up to the limit of accuracy in his system.

These instances give the first practical tools in his system. For example, the basic approach consists of a step from the First Instance to the Second, another step from there to the Third Instance and a thrust with a long step toward the opponent. In almost every chapter of his book, many of the "exercises" begins with an approach from an Instance to another.

As a short introduction, one can say that the First Instance is the boundary of the proper distance of the fencing. If the distance between the opponents is greater than the length of this Instance, they are treated as being "out of measure". The Second Instance determines the distance from where one can react to the movements of the opponent, and also the distance where the fencer can control his opponent's movement or blade in various ways. The Third Instance is the hall to the thrust; from here with a small movement one can reach the other with arm and sword held straight.




*Circle No.4 of Tabula I*

The length of the First Instance is in accordance with the length of the diameter, 24 ThU. The Second is 18.97 ThU (168.6 – 189.7 cm) moving along the side of the inscribed square (point G), or 19.69 ThU (175 – 196.9 cm) moving along the diameter (point E). The Third is 16.97 ThU (150.9 – 169.7 cm) moving along the side of the inscribed square (point N), and 16 ThU (142.2 – 160 cm) moving toward the diameter (point H). Thibault describes these distances very accurately.

But these are only the distances between the named points. To give the correct distances between the fencers, considering the positions of the feet and the body, he introduces a correction factor of 2.48 ThU (22.5 – 24.8 cm). Furthermore, it is important to describe the positions of the blades. To do this he determines the length of the perfect blade as 12 ThU. (Later, in the same chapter, he discusses it qualitatively and clarifies the relevancy of using blades with the proper length – not too short, not too long, 12 ThU).

The length of the blade and the arm together would be 20 ThU (177.8 – 200 cm), but Thibault corrects this value by subtracting 1 ThU, because he assumes that the holding of the sword (the fingers grip the ricasso, which is a part of the 12 ThU of the blade) shortens the 20 ThU length by 1 ThU. Hence, with the extension of the arm, one can reach 19 ThU (168.9 – 190 cm) with the sword. With this we get the following results.

From the First Instance one has to reach 24+2.48=26.48 ThU. It means that his movement has to be at least 26.48-19=7.48 ThU (66.5 – 74.8 cm) long. This also means that one can reach the other with one step (7.864 ThU), or by extremely bending his body. Beyond this one cannot reach his opponent without a full step.

From the Second Instance one has to reach 18.97+2.48=21.45 ThU from letter G, or 19.69+2.48=22.17 ThU from letter E. In these cases the movement of the blade has to be at least 2.45 ThU (21.8 – 24.5 cm) long standing on letter N or 3.17 ThU (28.2 – 31.7 cm) standing on letter E. To do this we do not need to step, bending the body is enough to reach the body of the other fencer.

From the Third Instance we have to reach 19.45 ThU (letter N), or 18.48 ThU (letter H), which means that from letter N we have to move only a little (0.45 ThU, which is approximately 4 – 4.5 cm), and from letter H we can reach the opponent with the extension of the arm.

Our calculations show exactly the same results, as they are based on simple Pythagorean theorem.

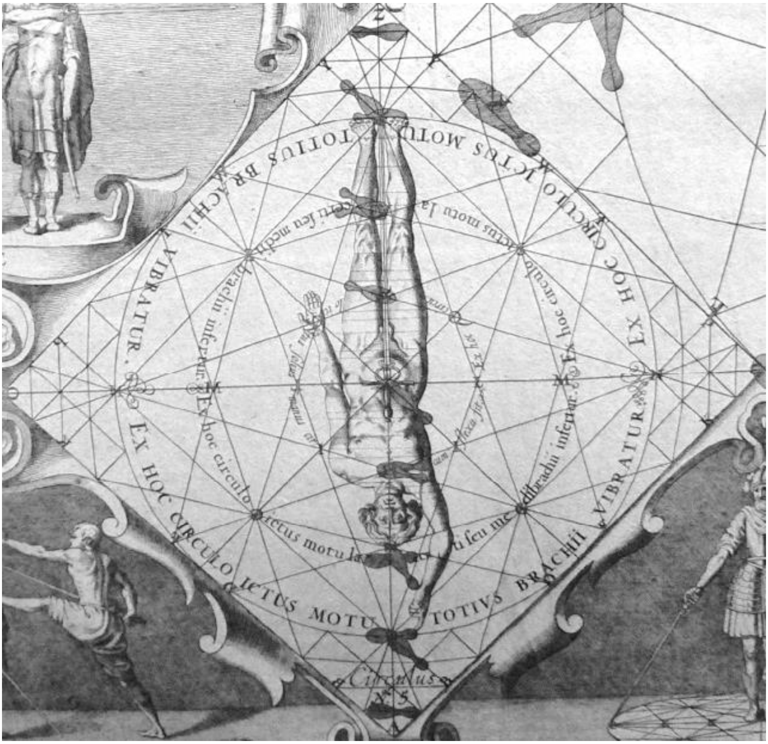
It is an interesting question why Thibault introduces the small correction factor as 2.48 ThU. To answer this we have to show another definition (or property) of the First Instance. Standing on this instance the blades of the adversaries are parallel, and the points reach the quillion of the others sword. Hence, the distance between the fencers bodies is equal to the length of one blade (12 ThU) minus the ricasso (0.879 ThU), plus the length of the two ricasso-s, and the two extended arms. It is altogether 26.879 ThU long. This result shows that Thibault's assumption about the correction factor is almost perfectly accurate, the difference is only a few centimeters, and even this can be credited to some small variations (the points do not touch the quillions, they are just very close, the grip is not entirely 1 ThU, and so on), which change when one stands on the different Instances. There are subtle variations described by Thibault in detail, but only qualitatively.

On the other hand the fact that Thibault chooses the correction factor to be 2.48 ThU leads to an important remark. He says the distance between the adversaries is not equal to the distance between the points, where their right feet are. The distance between them is greater (he refers to the half-length of the foot line), since the centers of the gravity of their body are not above the right leg. Moreover, the center of gravity is actually over the point between the feet, closer to the left. Although he gives no further data or calculations about this correction, it is a fact that in his system one begins the steps between the named Instances with the right foot and hence he has to relieve his right foot during movements, corresponding to the above results.

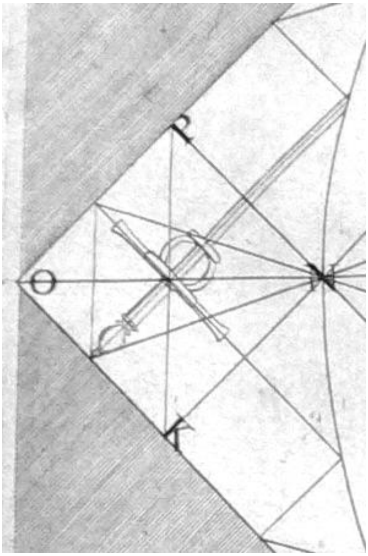
Note: At the end of the paragraph, Thibault collects the most important results into a small table. There is a typo in this table. The length of the Second Instance with correction is not 22.45, it is 21.45. When describing the details of the evaluation, the number is indicated correctly, this mistake occurs only in this table.

## 3. The proper measures of the sword

At the end of this chapter, after many basic ideas and theoretical approaches we would like to give some results (which are practically useful for fencers) for those who would like to follow Thibault's teachings. In the first and second chapter of his book he defines the proper measures of the sword.




*Circle No.5 of Tabula I*




*Part of Tabula II*

The length of the blade is determined in the first chapter as the half diameter of the Circle, hence it is 12 ThU (106.7 – 120 cm) long. He makes this statement clear from many points of view, and makes a stand for the proper length of the blade (not too short, not too long). In the second chapter he defines more measures. It is worth to mentioning that we only describe completely verifiable information, which is clearly defined in the text. The details of our calculations are described at the end of Appendix B.

From Tabula II one can see (and Thibault defines it in detail in the text) that the length of the grip including the pommel is equal to half the length of the side of the quadrangle, namely 1.757 ThU (15.6 – 17.6 cm). The length of the ricasso is the half of this distance, which is 0.879 ThU (7.8 – 8.8 cm). The length of the quillions is defined to be equal to the length of the foot line, which is 2.485 ThU (22.1 – 24.9 cm).

Thibault gives the detailed description of the pommel, gives some advice about the grip and determines the proper measures of the hanger as well. However, because these descriptions are not definitive we neglect to include them in this article, but we hope that we will be able to investigate these questions in the future.

There is one more piece of advice from us for those who would like to make or order a rapier, which Thibault would confirm as proper, and which fits to his system. Although he defines the ThU in two different ways, namely with the height of the human body with the arm held straight above (24 ThU), or with the distance between the soles and the navel (12 ThU), we suggest to do it another way. These definitions are extremely dependent on the actual parameters of the body and the details of the posture.

It should be more useful to derive the unit 1 ThU from the fact that the length of the arm is equal to 8 ThU, but Thibault does not specify how to measure this length (up to his amazing accuracy of 0.01 ThU). Although the length of the arm is more characteristic for a fencer, the most accurately measurable distance is the simple height of the body from the soles to the top of the head, which is equal to 19.69 ThU (175 – 196.9 cm). Of course, if one knows he has a non-average build, he can use all the above methods, and calculate an average for a more certain result.

After determining the proper length of 1 ThU in centimeters, one can evaluate every detail of the proper rapier using the above calculations.

## IV. ANTHROPOMETRICAL COMPARISONS

If we look at the human proportions in Thibault's system, it's easy to notice that he defined the measuring points of the human body to correspond with the various points of the Circle described in Section I.2.. These points, however, are very different from the modern-day anthropometric measuring point standards [WHO, 1995], which makes it difficult to compare anthropometric statistics and the body morphology derived from the Circle. Furthermore, Thibault only measured distances vertically on the sagittal plane, while in modern anthropometrics the circumference of the limbs and the waist is also important [WHO, 1995]. ThUs, the closest we can get to comparing the stature, distance and the length of key anatomical structures of the human body is to calculate the distance of several points in Thibault's system to match the modern anthropometrical measuring points.

For this reason, we selected the total standing height, the length of the upper arm and the length of the upper leg for comparative purposes (Table I.). The total upper arm length in Thibault's case is specified by calculating the distance between the center point (or the point of the navel) and Point E (which corresponds to the upper part of the shoulder) in ThU. The total upper leg length was calculated similarly from Point M (the perineum) and the average distance of Point S and T (the upper and lower part of the knee) which corresponds to the popliteal. Using the data mentioned before, the derived Thibaultian upper arm length is 4.566 ThU, while the derived Thibaultian upper leg length is 5,036 ThU. The authors defined these distances with the use of the *III. National Health and Nutrition Examination Survey* of the USA, which uses internationally, recognized anthropometric measuring points [*National Health and Nutrition Examination Survey III.*, 1988].

It is worth mentioning that in this Chapter we use the explicit value of the average standing height of men (according to the cited research), and hence the two examples (the 175 cm and the 196.9 cm tall fencer) from the first chapter is not relevant to these antrophometrical comparisons.

In our comparison we also included other sources from different periods in history, including Leonardo Da Vinci's Vitruvian Man, which is based on the works of the architect Vitruvius; and the Textbook of Plastic Anatomy, written by E. Harless in 1858 [Harless, 1858]. These sources were processed by Lon Kilgore of the University of the West of Scotland, whose calculations supplied the basis for this part of our study [Kilgore, 2012]. To compare these sets of anthropometric data with the Circle, first we converted the different results from centimeter to ThU by using the height as a standard (Table II./A). As described previously, we showed that the height of the fencer is 19.69 ThU, so we simply calculated the value of 1 ThU in cm from the height average of a dataset. After we defined the length of the ThU for each dataset, we could compare the Thibaultian calculated lengths described in the previous paragraph and the measured lengths from the studies used. This way we could show the differences between these values in ThU, in centimeters and as a percentage. Furthermore, we also examined the deviation between the different datasets used to examine if there are any major differences between them.

| | **Standing Height** | **Upper arm length** | **Upper leg length** |
|---|---|---|---|
| Thibault Derived Distances in ThU | 19.69 | 4.566 | 5.036 |
| Renaissance Estimated Measurements in cm (after Da Vinci, 1490) | 167.6 | 21 | 44.9 |
| Renaissance Estimated Measurements in ThU | 19.69 | 2.467 | 5.275 |
| Difference in percentage (Thibault's result is the reference) | X | - 8.745% | 0.997% |
| Difference in centimeter (Thibault's result is the reference) | X | - 14.656 | 1.671 |
| 19th century measurements in cm (after Harless, 1858) | 172.7 | 36.4 | 47.1 |
| 19th century measurements in ThU | 19.69 | 4.15 | 5.37 |
| Difference in percentage (Thibault's result Is the reference) | X | - 1.733% | 1.393% |
| Difference in centimeter (Thibault's result is the reference) | X | - 2.992 | 2.406 |
| Modern Day Average Measurements in cm | 178.45 | 39.65 | 42.85 |
| Modern Day Average Measurements in ThU | 19.69 | 4.375 | 4.728 |
| Difference in percentage (Thibault's result is the reference) | X | - 0.796% | - 1.282% |
| Difference in centimeter (Thibault's result is the reference) | X | - 1.42 | - 2.287 |

*Table II./A Comparison of the proportions of the human body in Thibault's system, the modern antromomethrical results and other historical models of the human body*

| | Standing Height | Upper arm length | Upper leg length |
|---|---|---|---|
| Difference in percentage between Modern and Renaissance values | X | 7.949% | - 2.279% |
| Difference in percentage between Modern and 19th century values | X | 0.937% | - 2.675% |
| Difference in percentage between Renaissance and 19th century values | X | - 7.012% | - 0.396% |

*Table II./B Comparison of modern anthropometrical results and the models of Da Vinci and Harless*

As the first table shows, there are small differences between Thibault's proportional representation and the proportional measures of other authors and statistics throughout history. The most prominent difference from Thibault's system is in the case of Da Vinci's Vitruvian Man's upper arm length, which is almost 9% shorter from the same value of Thibault (Table II./A.). But if we compare Da Vinci's drawing with the other authors beside Thibault, a similar discrepancy can be observed, ThUs it is likely that Da Vinci's work may be inaccurate in this case (Table II./B). If we look at the other datasets beside Da Vinci's drawing, most of these show very little deviation from the Thibaultian proportions. The 19th century measurements carried out by Harless in 1858 show less than 3% deviation from the Thibaultian derived lengths (Table II./A). Also, the data from the modern day anthropometrical survey shows even less deviation, which is closer to 2% (Table II./A). If we look at the deviation between the different datasets, one can also notice, that these datasets deviate just as much from each other than from Thibault, with the exception of Da Vinci's upper arm length, which shows large deviation of every data examined here (Table II./B).

These differences may be credited to the low sample size of the anthropometric studies compared here or even populational differences based on regional and temporal anthropometric variation. Furthermore, discrepancies can also be observed because the authors had to determine the distances corresponding to the modern anthropometric system, which in turn may be somewhat different from the points described by Thibault. This data may imply that Thibault's notions on human proportions are close to the truth, ThUs it seems that the Dutch master did not alter human proportions to fit his fencing system. For a more thorough study in the future it may be possible to compare other modern anthropometric measurement points with the Circle.

## Comparing the works of Thibault and Dürer from an anthropometrical standpoint

In the second chapter of *Academy of the Sword* Thibault compares his work to Albrecht Dürer's, who was considered as an authority on anatomy and human proportions in the Dutch master's time. The *Four Book on Human Proportions* was written by Dürer between 1512 and 1528 detailing various observations based on human proportions, physique, physiognomy and movement [Panofsky, 1955]. Thibault chose one of Dürer's original illustrations from these books, plotted his own circle on it and studied the differences between his own and Dürer's measuring points [Thibault, 1630]. From this composite drawing, he determined that his proportional calculations perfectly match Dürer's measurements for the most part, which proves the correctness of his thesis.

Thibault also emphasizes that the few differences between Dürer's measurements and his calculations come from errors that Dürer made because his system is man-made, while Thibault's is based on natural principles [Thibault, 1630]. He also discusses these errors in detail: According to Thibault the neck is too long, causing the shoulders, the armpits and the nipples to be depicted somewhat lower than where they should be; the buttocks and the knees are also too high, which is why there are some differences in the proportion of  the legs. The feet and hands are also much longer than in reality. This begs the question of whether Thibault's or Dürer's work is closer to reality regarding human proportions.

The previous chapter showed that the measurements derived from Thibault's Circle only slightly differ from several anthropometrical studies. This might imply that Thibault's system is closer to the natural proportions of man, than Dürer's. Also the differences between proportions described by the two authors are not really significant. Moreover, they also used local population to verify their work, ThUs both of them may be correct in this matter.  All in all, if we accept that Thibault's work is correct based on the conclusions of the previous chapter, either Thibault is closer to the natural proportions, or both Thibault and Dürer are correct considering the geographical and temporal distance between the two.

## V. CONCLUSIONS AND REMARKS

During the first phase of our work we wanted to investigate the possibility of describing Thibault's system with basic mathematical and physical methods. We hoped that it would give us the opportunity to create some analytic methods for other traditions of fencing. To get to this point we needed to describe the basic elements of the system, as well as hand look for a verification of the mathematics in Thibault's art.

Our first result on this path was the coordinate geometric description of the Circle, as the most important basic tool of the *Academy of the Sword*. With the derived expressions one can calculate any distances and angles in the Circle.

Furthermore, by comparing the results of modern anthropometry to Thibault's statements we have an approximation to determine how accurate his system is. To summarize the results of the calculations, and the mentioned comparison, we have found that Thibault's system is so precise that we could find important inaccuracies only at the level of 0.1 ThU, or up to 1 cm, after conversion.

Of course, we have found some differences of greater level, but it turned out that one can presume some ambiguities of definitions in these cases, or some subtle variations are summing up. On the other hand we have found that most of the author's statements are completely correct up to his limit of accuracy (0.01 ThU). These results indicate that the monumental mathematical background of Thibault's system is accurate enough to build our later models on this base. It is worth mentioning that our calculations indicate that this system is one of the most accurate among the historical systems we know of so far.

We also tried to give some practical information for those who study the fencing art of the *Academy of the Sword*, with providing some calculations about the length of the typical paces in the Circle, a detailed analysis of the three instances and the proper measurements of the blade according to Thibault's opinion.

There are two other main tools/methods/ideas in this system to be modeled. The first is the graduation/degraduation of the blades and the second is the system of Sentiment. To describe them, we plan to give a model of l'Attachment, the joining of the blades, in the future. We also plan to continue to compare the proportions of the human body in the Circle with modern anthropometric measures. This gives us the tools necessary to examine proper measurements on as many fencers of our time as possible, and adapt Thibault's system to the present.

## VI. ACKNOWLEDGEMENTS

We would like to thank Tamás Kiss for the diagrams, Attila Törcsvári for his suggestions and ideas which helped us significantly when planning our research. We are also thankful for the help of Ferenc Rádi, who checked our calculations. And we have to mention Szabolcs Waldmann, who introduced us the world of Thibault.

## VII. APPENDIX

In all the Appendices we use the following notations.

- We denote the points with big letters following the notations of Thibault himself. If there are two points denoted with the same letters, we distinguish them with subscripts: „l" denotes the points on the left half (with negative „x" coordinate value), and „r" denotes the point on the right side (with positive „x" coordinate value).

- We denote the lines with the letters of the defining points with a small line over them.

### 1. Points and lines

Here we give the detailed calculations for giving the coordinates of the named points in the Circle, and to give the equations of the lines of it [3].

We recall that the pole of our coordinate system is the centre of the circle, the 'x' axis points to the point O, the vertex of the circumscribed square on the right side of the picture of the Circle. The 'y' axis points to the point Z.

To give the coordinates of each point and the equations of each line in the Circle we need only the following two equations. The first is the equation of the circle with center (0,0) and with radius 12 ThU:

$$x^2 + y^2 = 12^2$$

The other one is the equation of a line defined by two points, namely $(x_1, y_1)$ and $(x_2, y_2)$:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

To determine the coordinates of the points we treat the equations of the intersecting lines as an equation system. Furthermore, if it is possible, we use the Pythagorean theorem as well.

It is worth mentioning that from now we do not write ThU, but our results are measured in this unit.

It would be redundant to evaluate everything separately, since there are many symmetries of the circle. In each step we give the equations/coordinates in the right upper section, and for the others we use these symmetries. During our evaluations we follow the pattern described in Chapter I.1.

### Step I. – The circle, the basic points and the first set of auxiliary points

$X = 12 \cdot (0,1)$

$N_r = 12 \cdot (1,0)$

$N_l = 12 \cdot (-1,0)$

$C = 12 \cdot (0,-1)$

$Z = 12\sqrt{2} \cdot (0,1) \approx 16.971 \cdot (0,1)$

$O_r = 12\sqrt{2} \cdot (1,0) \approx 16.971 \cdot (1,0)$

$O_l = 12\sqrt{2} \cdot (-1,0) \approx 16.971 \cdot (-1,0)$

$A = 12\sqrt{2} \cdot (0,-1) \approx 16.971 \cdot (0,-1)$

$$\acute{E}_{++} = \frac{12}{\sqrt{2}}(1,1) \approx 8.485 \cdot (1,1)$$

$$\acute{E}_{+-} = \frac{12}{\sqrt{2}}(1,-1) \approx 8.485 \cdot (1,-1)$$

$$\acute{E}_{-+} = \frac{12}{\sqrt{2}}(-1,1) \approx 8.485 \cdot (-1,1)$$

$$\acute{E}_{--} = \frac{12}{\sqrt{2}}(-1,-1) \approx 8.485 \cdot (-1,-1)$$

$$x^2 + y^2 = 12^2$$

### Step II. – Basic lines of the circumscribed and inscribed squares

1 / 1 $\overline{O_l O_r}: \quad y = 0$

1 / 2 $\overline{AZ}: \quad x = 0$

1 / 3 $\overline{\acute{E}_{--}\acute{E}_{++}}: \quad y = x$

1 / 4 $\overline{\acute{E}_{-+}\acute{E}_{+-}}: \quad y = -x$

2 / 1 $\overline{XN_r}: \quad y = -x + 12$

2 / 2 $\overline{N_l X}: \quad y = x + 12$

2 / 3 $\overline{CN_r}: \quad y = x - 12$

2 / 4 $\overline{N_l C}: \quad y = -x - 12$

3 / 1 $\overline{ZO_r}: \quad y = -x + 12\sqrt{2} \approx -x + 16.971$

3 / 2 $\overline{O_l Z}: \quad y = x + 12\sqrt{2} \approx x + 16.971$

3 / 3 $\overline{AO_r}: \quad y = x - 12\sqrt{2} \approx x - 16.971$

3 / 4 $\overline{O_l A}: \quad y = -x - 12\sqrt{2} \approx -x - 16.971$

**Step III. – New intersections**

$$Y_r: \quad \begin{Bmatrix} \overline{N_l X}: & y = x + 12 \\ \overline{ZO_r}: & y = -x + 12\sqrt{2} \end{Bmatrix} \quad \rightarrow \quad Y_r = 6 \cdot \left(\sqrt{2} - 1, \sqrt{2} + 1\right) \approx (2.485, \quad 14.485)$$

$$Y_l = 6 \cdot \left(-\sqrt{2} + 1, \sqrt{2} + 1\right) \approx (-2.485, \quad 14.485)$$

$$B_r = 6 \cdot \left(\sqrt{2} - 1, -\sqrt{2} - 1\right) \approx (2.485, \quad -14.485)$$

$$B_l = 6 \cdot \left(-\sqrt{2} + 1, -\sqrt{2} - 1\right) \approx (-2.485, \quad -14.485)$$

$$P_r: \quad \begin{Bmatrix} \overline{CN_r}: & y = x - 12 \\ \overline{ZO_r}: & y = -x + 12\sqrt{2} \end{Bmatrix} \quad \rightarrow \quad P_r = 6 \cdot \left(\sqrt{2} + 1, \sqrt{2} - 1\right) \approx (14.485, \quad 2.485)$$

$$P_l = 6 \cdot \left(-\sqrt{2} - 1, \sqrt{2} - 1\right) \approx (-14.485, \quad 2.485)$$

$$K_r = 6 \cdot \left(\sqrt{2} + 1, -\sqrt{2} + 1\right) \approx (14.485, \quad -2.485)$$

$$K_l = 6 \cdot \left(-\sqrt{2} - 1, -\sqrt{2} + 1\right) \approx (-14.485, \quad -2.485)$$

$$S_r: \quad \begin{Bmatrix} \overline{\acute{E}_{--}\acute{E}_{++}}: & y = x \\ \overline{XN_r}: & y = -x + 12 \end{Bmatrix} \quad \rightarrow \quad S_r = 6 \cdot (1,1)$$

$$S_l = 6 \cdot (-1,1)$$

$$G_r = 6 \cdot (1,-1)$$

$$G_l = 6 \cdot (-1,-1)$$

**Step IV. – New lines**

4 / 1 $\quad \overline{G_l X}: \quad \begin{Bmatrix} G_l = 6 \cdot (-1,-1) \\ X = 12 \cdot (0,1) \end{Bmatrix} \quad \rightarrow \quad y = 3x + 12$

4 / 2 $\quad \overline{XG_r}: \quad y = -3x + 12$

4 / 3 $\quad \overline{S_l C}: \quad y = -3x - 12$

4 / 4 $\quad \overline{CS_r}: \quad y = 3x - 12$

5 / 1 $\quad \overline{S_l N_r}: \quad \begin{Bmatrix} S_l = 6 \cdot (-1,1) \\ N_r = 12 \cdot (1,0) \end{Bmatrix} \quad \rightarrow \quad y = -\frac{1}{3}x + 4$

5 / 2 $\quad \overline{G_l N_r}: \quad y = \frac{1}{3}x - 4$

5 / 3 $\quad \overline{N_l S_r}: \quad y = \frac{1}{3}x + 4$

5 / 4 $\quad \overline{N_l G_r}: \quad y = -\frac{1}{3}x - 4$

**Step V. – Inner and outer intersections**

The outer intersections

$$W_r : \begin{cases} \overline{CS_r} : & y = 3x - 12 \\ \overline{ZO_r} : & y = -x + 12\sqrt{2} \end{cases} \rightarrow W_r = 3\cdot\left(\sqrt{2}+1, \quad 3\sqrt{2}-1\right) \approx (7.243, \quad 9.728)$$

$$W_l = 3\cdot\left(-\sqrt{2}-1, \quad 3\sqrt{2}-1\right) \approx (-7.243, \quad 9.728)$$

$$D_r = 3\cdot\left(\sqrt{2}+1, \quad -3\sqrt{2}+1\right) \approx (7.243, \quad -9.728)$$

$$D_l = 3\cdot\left(-\sqrt{2}-1, \quad -3\sqrt{2}+1\right) \approx (-7.243, \quad -9.728)$$

$$T_r = 3\cdot\left(3\sqrt{2}-1, \quad \sqrt{2}+1\right) \approx (9.728, \quad 7.243)$$

$$T_l = 3\cdot\left(-3\sqrt{2}+1, \quad \sqrt{2}+1\right) \approx (-9.728, \quad 7.243)$$

$$F_r = 3\cdot\left(3\sqrt{2}-1, \quad -\sqrt{2}-1\right) \approx (9.728, \quad -7.243)$$

$$F_l = 3\cdot\left(-3\sqrt{2}+1, \quad -\sqrt{2}-1\right) \approx (-9.728, \quad -7.243)$$

The inner intersections

$$R : \begin{cases} \overline{AZ} : & x = 0 \\ \overline{N_l S_r} : & y = \frac{1}{3}x + 4 \end{cases} \rightarrow R = 4\cdot(0,1)$$

$$H = 4\cdot(0,-1)$$

$$L_r : \begin{cases} \overline{O_l O_r} : & y = 0 \\ \overline{CS_r} : & y = 3x - 12 \end{cases} \rightarrow L_r = 4\cdot(1,0)$$

$$L_l = 4\cdot(-1,0)$$

$$Q_r : \begin{cases} \overline{\acute{E}_{--}\acute{E}_{++}} : & y = x \\ \overline{S_l N_r} : & y = -\frac{1}{3}x + 4 \end{cases} \rightarrow Q_r = 3\cdot(1,1)$$

$$Q_l = 3\cdot(-1,1)$$

$$I_r = 3\cdot(1,-1)$$

$$I_l = 3\cdot(-1,-1)$$

## Step VI. – Our last lines

6 / 1 $\overline{S_l W_r}$: $\left\{\begin{matrix} S_l = 6\cdot(-1,1) \\ W_r = 3\cdot\left(\sqrt{2}+1,\quad 3\sqrt{2}-1\right)\end{matrix}\right\} \rightarrow \quad y = \frac{3\sqrt{2}-3}{\sqrt{2}+3}x + \frac{24\sqrt{2}}{\sqrt{2}+3} \approx 0.282x + 7.67$

6 / 2 $\overline{W_l S_r}$: $y = -\frac{3\sqrt{2}-3}{\sqrt{2}+3}x + \frac{24\sqrt{2}}{\sqrt{2}+3} \approx -0.282x + 7.67$

6 / 3 $\overline{G_l D_r}$: $y = -\frac{3\sqrt{2}-3}{\sqrt{2}+3}x - \frac{24\sqrt{2}}{\sqrt{2}+3} \approx -0.282x - 7.67$

6 / 4 $\overline{D_l G_r}$: $y = \frac{3\sqrt{2}-3}{\sqrt{2}+3}x - \frac{24\sqrt{2}}{\sqrt{2}+3} \approx 0.282x - 7.67$

7 / 1 $\overline{S_r F_r}$: $\left\{\begin{matrix} S_r = 6\cdot(1,1) \\ F_r = 3\cdot\left(3\sqrt{2}-1,\quad -\sqrt{2}-1\right)\end{matrix}\right\} \rightarrow \quad y = -\frac{\sqrt{2}+3}{3\sqrt{2}-3}x + \frac{8\sqrt{2}}{\sqrt{2}-1} \approx -3.552x + 27.313$

7 / 2 $\overline{F_l S_l}$: $y = \frac{\sqrt{2}+3}{3\sqrt{2}-3}x + \frac{8\sqrt{2}}{\sqrt{2}-1} \approx 3.552x + 27.313$

7 / 3 $\overline{T_r G_r}$: $y = \frac{\sqrt{2}+3}{3\sqrt{2}-3}x - \frac{8\sqrt{2}}{\sqrt{2}-1} \approx 3.552x - 27.313$

7 / 4 $\overline{T_l G_l}$: $y = -\frac{\sqrt{2}+3}{3\sqrt{2}-3}x - \frac{8\sqrt{2}}{\sqrt{2}-1} \approx -3.552x - 27.313$

## Step VII. – Our last points

$V: \left\{\begin{matrix} \overline{AZ}: \quad x = 0 \\ \overline{S_l W_r}: \quad y = \frac{3\sqrt{2}-3}{\sqrt{2}+3}x + \frac{24\sqrt{2}}{\sqrt{2}+3}\end{matrix}\right\} \rightarrow \quad V = \frac{24\sqrt{2}}{\sqrt{2}+3}(0,1) \approx 7.67\cdot(0,1)$

$E = \frac{24\sqrt{2}}{\sqrt{2}+3}(0,-1) \approx 7.67\cdot(0,-1)$

$M_r = \frac{24\sqrt{2}}{\sqrt{2}+3}(1,0) \approx 7.67\cdot(1,0)$

$M_l = \frac{24\sqrt{2}}{\sqrt{2}+3}(-1,0) \approx 7.67\cdot(-1,0)$

## Step VIII. – Auxiliary lines and the foot line

Medians of the quadrangles

8 / 1 $\overline{Y_l Y_r}$: $y = 6\cdot\left(\sqrt{2}+1\right) \approx 14.485$

8 / 2 $\overline{B_l B_r}$: $y = -6\cdot\left(\sqrt{2}+1\right) \approx -14.485$

8 / 3 $\overline{P_r K_r}$: $x = 6\cdot\left(\sqrt{2}+1\right) \approx 14.485$

8 / 4 $\overline{P_l K_l}$: $x = -6\cdot\left(\sqrt{2}+1\right) \approx -14.485$

The foot line for the quadrangles $XY_rZY_l$ and $AB_rCB_l$. The other two are similar. They can be evaluated from symmetries, but we do not need them.

$$p/1 \quad y_{pXYZX} = 3\cdot\left(3\sqrt{2}+1\right) \approx 15.728$$

$$p/2 \quad y_{pABCB} = -3\cdot\left(3\sqrt{2}+1\right) \approx -15.728$$

At last we give the equations of the half-way lines of these quadrangles as

$$h/1+ \quad y = x + 6\cdot\left(\sqrt{2}+1\right) \approx x + 14.485$$

$$h/1- \quad y = -x + 6\cdot\left(\sqrt{2}+1\right) \approx -x + 14.485$$

$$h/2+ \quad y = x - 6\cdot\left(\sqrt{2}+1\right) \approx x - 14.485$$

$$h/2- \quad y = -x - 6\cdot\left(\sqrt{2}+1\right) \approx -x - 14.485$$

The half-way lines of the other two quadrangle are lying on these lines too, but with different end points.

## 2. The measures of the human body

These measures are determined in two different ways; hence we have to introduce a new notation. Some of these measures are determined by points located on the vertical diameter, but most of them are determined by small lines. These small lines are defined by typical points or intersections of the other main lines. To distinguish the points and the lines (The points are also identified by a single capital letter) we will denote these lines with a small line above the letter.

Besides the expressions of Appendix A, we need the equation which determines the distance between two points, namely $A=(A_x , A_y)$ and $B=(B_x , B_y)$.

$$\left|\overline{AB}\right| = \sqrt{\left(B_x - A_x\right)^2 + \left(B_y - A_y\right)^2}$$

For example the height of the body with arm held straight above is

$$\left|\overline{CX}\right| = y_X - y_C = 12 - (-12) = 24,$$

or the height of the body from soles to the top of the head is

$$\left|\overline{CV}\right| = y_V - y_C \approx 7.67 - (-12) = 19.67.$$

From now on we only give the ‘y’ coordinates of the defining points or the small lines describing the position of the named part of the body.

Top of the face – line A

$$\begin{cases} \overline{G_l X}: & y = 3x + 12 \\ \overline{W_r S_l}: & y = \dfrac{3\sqrt{2}-3}{\sqrt{2}+3}x + \dfrac{24\sqrt{2}}{\sqrt{2}+3} \approx 0.282x + 7.67 \end{cases} \rightarrow \quad y_{\bar{A}} = 3 \cdot \left(\sqrt{2}+1\right) \approx 7.243$$

End of nose – line B

$$y_{\bar{B}} = y_S = 6$$

Chin – line C

$$\begin{cases} \overline{T_l G_l}: & y = -\dfrac{\sqrt{2}+3}{3\sqrt{2}-3}x - \dfrac{8\sqrt{2}}{\sqrt{2}-1} \approx -3.552x - 27.313 \\ \overline{W_r S_l}: & y = \dfrac{3\sqrt{2}-3}{\sqrt{2}+3}x + \dfrac{24\sqrt{2}}{\sqrt{2}+3} \approx 0.282x + 7.67 \end{cases} \rightarrow \quad y_{\bar{C}} = \dfrac{24\left(3\sqrt{2}-2\right)}{19-6\sqrt{2}} \approx 5.119$$

Adam’s apple – line D

$$\begin{cases} \overline{G_l X}: & y = 3x + 12 \\ \overline{S_l N_r}: & y = -\dfrac{1}{3}x + 4 \end{cases} \rightarrow \quad y_{\bar{D}} = 4.8$$

Top of the shoulders – line E

$$\begin{cases} circle: & x^2 + y^2 = 12^2 \\ \overline{W_r S_l}: & y = \dfrac{3\sqrt{2}-3}{\sqrt{2}+3}x + \dfrac{24\sqrt{2}}{\sqrt{2}+3} \approx 0.282x + 7.67 \end{cases} \rightarrow \quad y_{\bar{E}} \approx 4.566$$

Top of the chest – point R

$$y_R = 4$$

Armpits – line F

$$\begin{cases} \overline{N_l X}: & y = x + 12 \\ \overline{T_l G_l}: & y = -\dfrac{\sqrt{2}+3}{3\sqrt{2}-3}x - \dfrac{8\sqrt{2}}{\sqrt{2}-1} \approx -3.552x - 27.313 \end{cases} \rightarrow \quad y_{\bar{F}} = \dfrac{3 \cdot \left(3\sqrt{2}-2\right)}{2} \approx 3.364$$

Nipples – line G

$$y_{\bar{G}} = y_Q = 3$$

Middle of the chest – line H

$$\left\{\begin{array}{ll} \overline{S_l C}: & y = -3x - 12 \\ \overline{N_l S_r}: & y = \frac{1}{3}x + 4 \end{array}\right\} \rightarrow \quad y_{\bar{H}} = 2.4$$

Breastbone – line I

$$\left\{\begin{array}{ll} \overline{F_l S_l}: & y = \frac{\sqrt{2}+3}{3\sqrt{2}-3}x + \frac{8\sqrt{2}}{\sqrt{2}-1} \approx 3.552x + 27.313 \\ \overline{N_l S_r}: & y = \frac{1}{3}x + 4 \end{array}\right\} \rightarrow \quad y_{\bar{I}} = 3 - \sqrt{2} \approx 1.586$$

Floating ribs and diaphragm – line K

$$\left\{\begin{array}{ll} \overline{T_l G_l}: & y = -\frac{\sqrt{2}+3}{3\sqrt{2}-3}x - \frac{8\sqrt{2}}{\sqrt{2}-1} \approx -3.552x - 27.313 \\ \overline{N_l S_r}: & y = \frac{1}{3}x + 4 \end{array}\right\} \rightarrow \quad y_{\bar{K}} = \frac{2\cdot\left(3-\sqrt{2}\right)}{\sqrt{2}+1} \approx 1.314$$

From symmetry properties we can calculate the following:

Top of the hipbone – line L
Perineum – line M
Penis – line N
Anus – line O
Top of the thigh – line P
Greatest thickness of the thigh – point H
Hollow of the thigh – line Q
Bottom of the thigh – line R
Top of the knee – line S
Bottom of the knee – line T
Top of the shin – point E

Top of the thick part of the calf on the inside – line V

$$\left.\begin{cases} \overline{S_l C}: & y = -3x - 12 \\ \overline{D_l G_r}: & y = \dfrac{3\sqrt{2}-3}{\sqrt{2}+3}x - \dfrac{24\sqrt{2}}{\sqrt{2}+3} \approx 0.282x - 7.67 \end{cases}\right\} \rightarrow \quad y_{\overline{V}} = \frac{6\cdot\left(1-3\sqrt{2}\right)}{\sqrt{2}+1} \approx -8.059$$

Top of the thick part of the calf on the outside – line W

$$\left.\begin{cases} \overline{N_l C}: & y = -x - 12 \\ \overline{D_l G_r}: & y = \dfrac{3\sqrt{2}-3}{\sqrt{2}+3}x - \dfrac{24\sqrt{2}}{\sqrt{2}+3} \approx 0.282x - 7.67 \end{cases}\right\} \rightarrow \quad y_{\overline{W}} = \frac{3\cdot\left(3\sqrt{2}-10\right)}{2} \approx -8.636$$

Bottom of the thick part of the calf on the inside – line X

$$\left.\begin{cases} \overline{T_l G_l}: & y = -\dfrac{\sqrt{2}+3}{3\sqrt{2}-3}x - \dfrac{8\sqrt{2}}{\sqrt{2}-1} \approx -3.552x - 27.313 \\ \overline{D_l G_r}: & y = \dfrac{3\sqrt{2}-3}{\sqrt{2}+3}x - \dfrac{24\sqrt{2}}{\sqrt{2}+3} \approx 0.282x - 7.67 \end{cases}\right\} \rightarrow \quad y_{\overline{X}} = -\frac{192}{38-12\sqrt{2}} \approx -9.13$$

Bottom of the shin – line Y

$$y_{\overline{Y}} = y_D \approx -9.728$$

Ankle – line Z

$$\left.\begin{cases} circle: & x^2 + y^2 = 12^2 \\ \overline{T_l G_l}: & y = -\dfrac{\sqrt{2}+3}{3\sqrt{2}-3}x - \dfrac{8\sqrt{2}}{\sqrt{2}-1} \approx -3.552x - 27.313 \end{cases}\right\} \rightarrow \quad y_{\overline{Z}} \approx -11.11$$

Soles of the feet – point C

$$y_C = -y_X = -12$$

Here we have collected our results in a simple form:

$$y_X = 12$$
$$y_V \approx 7.67$$
$$y_{\bar{A}} \approx 7.243$$
$$y_{\bar{B}} = 6$$
$$y_{\bar{C}} \approx 5.119$$
$$y_{\bar{D}} = 4.8$$
$$y_{\bar{E}} \approx 4.566$$
$$y_R = 4$$
$$y_{\bar{F}} \approx 3.364$$
$$y_{\bar{G}} = 3$$
$$y_{\bar{H}} = 2.4$$
$$y_{\bar{I}} \approx 1.586$$
$$y_{\bar{K}} \approx 1.314$$
$$y_{center} = 0$$
$$y_{\bar{L}} \approx -1.314$$
$$y_{\bar{M}} \approx -1.586$$
$$y_{\bar{N}} = -2.4$$
$$y_{\bar{O}} = -3$$
$$y_{\bar{P}} \approx -3.364$$
$$y_H = -4$$
$$y_{\bar{Q}} = -4.8$$
$$y_{\bar{R}} \approx -5.119$$
$$y_{\bar{S}} = -6$$
$$y_{\bar{T}} \approx -7.243$$
$$y_E \approx -7.67$$
$$y_{\bar{V}} \approx -8.059$$
$$y_{\bar{W}} \approx -8.636$$
$$y_{\bar{X}} \approx -9.13$$
$$y_{\bar{Y}} \approx -9.728$$
$$y_{\bar{Z}} \approx -11.11$$
$$y_C = -12$$

To complete our results we give the calculations according to the length of the foot line, and the other lines in the quadrangle.

The length of the diagonal of the $AB_rCB_l$ quadrangle is

$$l_{diameter} = \left|\overline{B_l B_r}\right| = 12 \cdot \left(\sqrt{2} - 1\right) \approx 4.971$$

With the use of it we get the following result for the foot line

$$l_{footline} = \frac{l_{diameter}}{2} = 6 \cdot \left(\sqrt{2} - 1\right) \approx 2.485 \cdot$$

The length of the side of the quadrangle is

$$l_{side} = \frac{l_{diameter}}{\sqrt{2}} = 6 \cdot \left(2 - \sqrt{2}\right) \approx 3.515$$

and we need the half and the quarter of this length, and these are

$$l_{half} = \frac{l_{side}}{2} = 3 \cdot \left(2 - \sqrt{2}\right) \approx 1.757$$

$$l_{quarter} = \frac{l_{side}}{4} = 3 - 1.5 \cdot \sqrt{2} \approx 0.879 \quad .$$

## 3. Evaluations to determine the length of the steps

The full length of 4 paces according to the soles denoted by '1' is:

$$x_{O_j} - x_{O_b} = 24\sqrt{2} \approx 33.941$$

The real length of 4 paces (the positions of the soles are taken into account) is:

$$l_{4pace} = 18\sqrt{2} + 6 \approx 31.456$$

Hence, one step according to the '1' soles is

$$l_{pace,1} = \frac{9\sqrt{2} + 3}{2} \approx 7.864$$

With this, the coordinates of the heels at each step are:

$$a_{heel0} = O_j = 12\sqrt{2}\cdot(1,0) \approx 16.9706\cdot(1,0)$$
$$a_{heel1} = \frac{15\sqrt{2}-3}{2}\cdot(1,0) \approx 9.1066\cdot(1,0)$$
$$a_{heel2} = \left(3\sqrt{2}-3\right)\cdot(1,0) \approx 1.2426\cdot(1,0)$$
$$a_{heel3} = \frac{-3\sqrt{2}-9}{2}\cdot(1,0) \approx -6.6213\cdot(1,0)$$
$$a_{heel4} = \left(-6\sqrt{2}-6\right)\cdot(1,0) \approx -14.4853\cdot(1,0)$$

The distance between the heels and the points M are:

$$a_{heel1} - M_r = \frac{21-6\sqrt{2}}{2\sqrt{2}+6}\cdot(1,0) \approx 1.4175\cdot(1,0)$$
$$a_{heel3} - M_l = \frac{30\sqrt{2}-33}{2\sqrt{2}+6}\cdot(1,0) \approx 1.068\cdot(1,0)$$

The length of 3 steps is:

$$3l_{pace,1} = \frac{27\sqrt{2}+9}{2} \approx 23.592$$

or with the inclusion of the foot line:

$$3l_{pace,1} + l_{footline} = \frac{39\sqrt{2}-3}{2} \approx 26.077$$

The length of one pace, according to the soles denoted with '6' is:

$$l_{pace,2} = \frac{\left|\overline{N_j C}\right|}{2} = 6\sqrt{2} \approx 8.485$$

The length of one pace according to the soles denoted with '6' with the heels as reference points is:

$$l_{pace,2heel} = \sqrt{\left(l_{pace,2} - \frac{l_{footline}}{2} + \frac{l_{footline}}{2}\sin(45^\circ)\right)^2 + \left(\frac{l_{footline}}{2}\cos(45^\circ)\right)^2} \approx 8.169$$

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